SCHEMES FOR COMPUTING PERFORMANCE PARAMETERS OF FIBER OPTIC GYROSCOPES

CROSS-REFERENCE TO RELATED APPLICATION

This application claims the priority of U.S. provisional Patent Application Serial No. 60/442,634 (by Humphrey, filed January 24, 2003, and entitled "SCHEMES FOR COMPUTING PERFORMANCE PARAMETERS OF FIBER OPTIC GYROSCOPES").

Background

(1) Field

The present disclosure relates to schemes for computing performance parameters of fiber optic gyroscopes (FOGs) using closed-loop transfer functions.

(2) Description of Related Art

A FOG is a device that can detect rotation in a variety of applications, including navigation and stabilization schemes. Generally, a FOG can include an optical subsystem and an electrical subsystem. The optical and electrical subsystems can provide inputs to each other.

A FOG can be characterized by a variety of performance parameters, including an operating frequency and a bandwidth. Generally, schemes for computing FOG performance parameters separately model FOG optical and electrical subsystems with two open-loop systems. Since FOGs can operate with their optical and electrical subsystems in a closed-loop configuration, however, meaningful conclusions cannot be reliably provided by two open-loop systems.

Summary

Schemes for computing performance parameters of FOGs using closed-

loop transfer functions are described herein.

A method for computing a performance parameter of a FOG is described herein. In one embodiment, the method may include providing a closed-loop transfer function based on optical components and electrical components of the FOG; based on the transfer function, determining a relationship between the performance parameter and at least one physical parameter associated with at least one component of the FOG; and, based on the relationship, computing the performance parameter.

In one aspect, providing may include providing a feedforward component representing at least one FOG optical component and at least one FOG electronic component; and, providing a feedback component representing at least one FOG optical component and at least one FOG electronics component.

In one aspect, providing a feedforward component may include representing, in the feedforward component, at least one noise component.

In one aspect, providing a feedforward component may include representing, in the feedforward component, at least one disturbance, wherein the at least one disturbance is based on at least one of: an optical power noise, a shot noise, a preamplifier current noise, a preamplifier thermal noise, a preamplifier voltage noise, and an analog-to-digital converter (ADC) quantization noise.

In one aspect, providing a feedforward component may include representing, in the feedforward component, at least one of: a phase modulator, a photodetector and an associated preamplifier, a filter, an ADC, and a sampler.

In one aspect, representing the phase modulator may include representing the phase modulator based on an optical power of a light beam propagating

through a fiber-optic coil and an operating phase bias.

In one aspect, representing the phase modulator may include representing the phase modulator based on a product of the optical power and a sinusoidal function of the operating phase bias.

In one aspect, representing the photodetector and the associated preamplifier may include representing the photodetector and the associated preamplifier based on a photodetector scale factor, a preamplifier impedance, and a preamplifier gain.

In one aspect, representing the photodetector and the associated preamplifier may include representing the photodetector and the associated preamplifier based on a product of the photodetector scale factor, the preamplifier impedance, and the preamplifier gain.

In one aspect, representing the filter may include representing the filter as a gain in voltage after the photodetector and associated preamplifier and before the ADC.

In one aspect, representing the ADC may include representing the ADC as a gain based on the number of bits in the ADC.

In one aspect, providing a feedback component may include representing, in the feedback component, at least one of: sampler, a truncator, a digital-to-analog converter (DAC), a phase modulator, and a fiber-optic coil.

In one aspect, representing the truncator may include representing the truncator as a digital truncation gain.

In one aspect, representing the DAC may include representing the DAC as a gain based on the number of bits in the DAC.

In one aspect, representing the phase modulator may include representing the phase modulator as a scale factor.

In one aspect, representing the fiber-optic coil may include representing the fiber-optic coil as a time delay.

In one aspect, representing the fiber-optic coil may include representing the fiber-optic coil based on a transit time for a light beam to propagate through the fiber-optic coil.

In one aspect, determining a relationship may include, based on the transfer function, determining a relationship between the performance parameter and at least one physical parameter associated with at least one component of the FOG, wherein the at least one physical parameter includes at least one of:

an optical power of a light beam propagating through a fiber-optic coil, an operating phase bias, a photodetector scale factor, a preamplifier impedance, a preamplifier gain, a filter gain, an ADC gain, a digital truncation gain, a DAC gain, a transit time for a light beam to propagate through the fiber-optic coil, and a phase modulator scale factor.

In one aspect, computing may include providing an input based on a rate of rotation of a fiber-optic coil and a scale factor, the scale factor including a wavelength of a light beam propagating through the coil, a coil length, and a coil diameter.

In one aspect, computing may include computing a performance parameter including at least one of a bandwidth, a coefficient of random walk, an operating frequency, and a power spectral density of noise.

In one embodiment, the method may further include providing a value of a

performance parameter and determining at least one value associated with the at least one physical parameter for which the computed performance parameter will have the value.

In one aspect, determining the at least one value may include providing at least one initial value associated with the at least one physical parameter; based on the relationship and the at least one initial value, computing the performance parameter; and, based on a difference between the computed performance parameter and the value, iteratively adjusting at least one value associated with the at least one physical parameter and iteratively computing the performance parameter.

In one embodiment, the method may further include providing a first value of a first performance parameter; providing a second value of a second performance parameter; and, determining at least one value associated with the at least one physical parameter for which the computed first performance parameter will approach the first value and the computed second performance parameter will approach the second value.

In one aspect, determining at least one value may include providing at least one initial value associated with the at least one physical parameter; based on the corresponding relationship and the at least one initial value, computing the first performance parameter and the second performance parameter; and, based on a difference between at least one of the first value and the computed first performance parameter and the second value and the computed second performance parameter, iteratively adjusting at least one value associated with the at least one physical parameter and iteratively computing the first performance

parameter and the second performance parameter.

A processor program for computing a performance parameter of a fiber-optic gyroscope (FOG) is described herein. In one embodiment, the processor program may be stored on a processor-readable medium and may include instructions to cause a processor to receive a closed-loop transfer function based on optical components and electrical components of the FOG; based on the transfer function, determine a relationship between the performance parameter and at least one physical parameter associated with at least one component of the FOG; and, based on the relationship, computing the performance parameter.

In one aspect, the instructions to compute may include instructions to compute a performance parameter including at least one of a bandwidth, a coefficient of random walk, an operating frequency, and a power spectral density of noise.

In one embodiment, the processor program may also include instructions to receive a value of a performance parameter, and determine at least one value associated with the at least one physical parameter for which the computed performance parameter will have the value.

In one aspect, the instructions to determine may include instructions to receive at least one initial value associated with the at least one physical parameter; based on the relationship and the at least one initial value, compute the performance parameter; and, based on a difference between the computed performance parameter and the value, iteratively adjust at least one value associated with the at least one physical parameter and iteratively compute the performance parameter.

In one embodiment, the processor program may also include instructions to receive a first value of a first performance parameter; receive a second value of a second performance parameter; and, determine at least one value associated with the at least one physical parameter for which the computed first performance parameter will approach the first value and the computed second performance parameter will approach the second value.

In one aspect, the instructions to determine may include instructions to receive at least one initial value associated with the at least one physical parameter; based on the corresponding relationship and the at least one initial value, compute the first performance parameter and the second performance parameter; and, based on a difference between at least one of the first value and the computed first performance parameter, and the second value and the computed second performance parameter, iteratively adjust at least one value associated with the at least one physical parameter and iteratively compute the first performance parameter and the second performance parameter.

These and other features of the schemes for computing performance parameters of FOGs described herein may be more fully understood by referring to the following detailed description and accompanying drawings.

Brief Description of the Drawings

Fig. 1 is a block diagram of an exemplary closed-loop transfer function for a FOG.

Fig. 2 is a block diagram of an exemplary feedforward component of the closed-loop transfer function shown in Fig. 2

Fig. 3 schematically illustrates a prior-art FOG.

Detailed Description

Certain exemplary embodiments will now be described to provide an overall understanding of the schemes for computing performance parameters of FOGs described herein. One or more examples of the exemplary embodiments are shown in the drawings.

Those of ordinary skill in the art will understand that the schemes for computing performance parameters of FOGS described herein can be adapted and modified to provide devices, methods, schemes, and systems for other applications, and that other additions and modifications can be made to the schemes described herein without departing from the scope of the present disclosure. For example, components, features, modules, and/or aspects of the exemplary embodiments can be combined, separated, interchanged, and/or rearranged to generate other embodiments. Such modifications and variations are intended to be included within the scope of the present disclosure.

Generally, the exemplary schemes described herein include a closed-loop representation of FOG optical subsystem components and FOG electrical subsystem components to compute performance parameters for FOGs. In one embodiment, a closed-loop transfer function can be used to determine a relationship between a FOG performance parameter and physical parameter(s) associated with FOG component(s). The relationship can be used to determine value(s) of the physical parameter(s) for which the performance parameter will approach a performance parameter value.

Fig. 3 schematically illustrates a prior-art FOG. FOGs are well known and may be understood by referring to the disclosures of U.S. Patent Nos. 4,705,399 to

Graindorge et al. and 5,337,142 to Lefevre et al., the contents of which patents are expressly incorporated by reference herein.

As shown in Fig. 3, FOG 10 may include an optical subsystem 12 and an electrical subsystem 14. Optical subsystem 12 may include a light source 22, a beam splitter 24, a phase modulator 26, and an optical waveguide 28. Electrical subsystem 14 may include a signal digitizer 30 and a demodulator 32. Optical subsystem 12 can provide a signal 16 to electrical subsystem 14, and electrical subsystem 14 can provide a feedback signal 18 to optical subsystem 12. Electrical subsystem 14 can also provide a signal 20 to an application. FOG components 22, 24, 26, 28, 30, and 32 may be connected by optical and/or electrical connection(s) and may communicate with component(s) other than those illustrated.

Operation of FOG 10 may be briefly understood in the following manner. Light source 22 can provide a light signal 15 to beam splitter 24, and beam splitter 24 can split the light signal into two light signals that travel in opposite directions 34, 36 along an optical path defined by optical waveguide 28. Beam splitter 24 can receive the two light signals exiting from optical waveguide 28, combine the two light signals, and provide the combined light signal 16 to signal digitizer 30. Based on the combined light signal 16, signal digitizer 30 can produce an output signal proportional to a phase difference between the two light signals exiting the optical waveguide 28. According to the well known Sagnac effect, this phase difference can be used to measure a rate of rotation of the optical waveguide 28.

A variety of schemes for adjusting the operating point of a FOG 10 are available. Generally, these schemes superimpose artificial phase differences on the two light signals 34, 36 counterpropagating in the optical waveguide 28. In

these schemes, the output from the signal digitizer 30 can be provided to the demodulator 32, and the demodulator 32 can provide a feedback signal 18 to phase modulator 26 to modulate the relative phases of the counterpropagating light beams.

Fig. 1 is a block diagram of an exemplary closed-loop transfer function for FOG 10. As shown in Fig. 1, the transfer function 100 may include an input 110, a summing point 120, a feedforward component 130, a feedback component 140, and a branch point 150. Input 110 and feedback component 140 may be provided to positive and negative terminals 122, 124 of summing point 120, respectively. As described herein, transfer function 100 may be used to compute an operating frequency and bandwidth of FOG 10. Appendices 1-5 include features of transfer function 100 described herein.

Generally, input 110 may be based on a rate of rotation of an optical waveguide 28 and a scale factor. Input 110 may be based on a product or the rate of rotation and the scale factor. In one embodiment, the scale factor may include a wavelength of light propagating through the optical waveguide 28, an optical path length of the optical waveguide 28, and a diameter of the optical waveguide 28. The scale factor may be associated with the well known Sagnac scale factor. For example, in one embodiment of FOG 10, an optical waveguide 28 may include a coil of optical fiber wound on a spool-type structure, such as a bobbin, and a light source 22 that can be, for example, a superluminescent diode (SLD). In such an embodiment, the input 110 may be represented as the product

(Eq. 1) $$\Omega K_s = \frac{\Omega(2\pi LD)}{\lambda}$$

where Q is the rate of rotation of the coil, K_s is the well known Sagnac scale factor, L is the length of the coil, D is the diameter of the coil, λ is the wavelength of light emitted by the SLD, and c is the speed of light in vacuo.

Feedforward component 130 may include representations of at least one FOG optical component and at least one FOG electrical component. As shown in Fig. 1, feedforward component 130 may include a representation 132 of a phase modulator 26. In one embodiment, phase modulator 26 may be represented based on an optical power of light emitted by light source 22 and an operating phase bias of FOG 10. An operating phase bias can refer to a phase bias applied to counterpropagating light beams 34, 36 in optical waveguide 28 to displace the operating point of FOG 10. In one embodiment, the phase modulator 26 may be represented based on a product of the optical power and a sinusoidal function of the operating phase bias. For example, the phase modulator may be based on the product

(Eq. 2) $$K_I = I_o \sin(\phi_b),$$

where I_o is the optical power of light source 22 and ϕ_b is the operating phase bias of FOG 10.

Feedforward component 130 may also include a representation 134 of a signal digitizer 30. Generally, a signal digitizer 30 may include a light detector, an analog-to-digital converter (ADC) , filter(s), and other processing component(s). A variety of signal digitizers may be represented based on schemes described herein.

In one embodiment, the signal digitizer 30 may be represented as including

a photodetector and an associated preamplifier 135, a filter 136, an ADC 137, and a sampler 138. The photodetector and associated preamplifier 135 may be represented based on a photodetector scale factor R_d, a preamplifier impedance R_f and a preamplifier gain G_e. The photodetector scale factor R_d may represent a scale factor between an input optical power and an output analog signal, e.g. current or voltage. In one embodiment, the photodetector and associated preamplifier 135 may be represented based on the product of the photodetector scale factor R_d, the preamplifier impedance R_f, and the preamplifier gain G_e. The ADC 137 may be represented as a gain based on a number of bits b in the ADC 137. In one embodiment, the ADC 137 may be represented as a gain based on the power 2^{b-1}. In one embodiment, the filter 136 may be represented as a gain G_f in voltage after the photodetector and associated preamplifier 135 and before the ADC 137. The sampler 138 may be represented as a sampler for analog-to-digital conversion. Accordingly, in one embodiment, the signal digitizer 30 may be represented based on the product

(Eq. 3) $$R_d R_f G_e G_f\, 2^{b-1}.$$

Feedback component 140 may include representations of at least one FOG optical component and at least one FOG electrical component. Feedback component 140 may include a representation 142 of a demodulator 32. Generally, a demodulator 32 may include a sampler, a truncator, a digital-to-analog converter (DAC), and other processing component(s). A variety of demodulators may be represented based on schemes described herein.

In one embodiment, the demodulator 32 may be represented as including a sampler 143, a truncator 144, and a DAC 145. The sampler 143 may be

represented as a sampler for digital-to-analog conversion. The truncator 144 may be represented as a digital truncation gain G_D that occurs after the sampler 143 and before the DAC 145. In one embodiment, the digital truncation gain G_D may be based on the number of bits d′ in the sampler 143 and the number of bits d in the DAC 145. For example, the digital truncation gain G_D may be based on the power $2^{d-d'}$. The DAC 145 may be represented as a gain based on the number of bits d in the DAC 145. In one embodiment, the DAC 145 may be represented as a gain based on the power 2^{2-d}. Accordingly, in one embodiment, the demodulator 30 may be represented based on the product

(Eq. 4) $$2^{d-d'}2^{2-d}=2^{2-d'}.$$

Feedback component 140 may include a representation 146 of a phase modulator 26. In one embodiment, phase modulator 26 may be represented based on a phase modulator scale factor K_{pm}. The phase modulator scale factor K_{pm} may represent a scale factor between an input analog signal, e.g. current or voltage, and an output angular measure.

Feedback component 140 may also include a representation 148 of an optical waveguide 28. In one embodiment, the optical waveguide 28 may be represented as a time delay. The optical waveguide 28 may be represented as a transit time τ for light to propagate through optical waveguide 28. For example, as previously described, an optical waveguide 28 may include a coil of optical fiber. In such an embodiment, the optical waveguide 28 may be represented based on a transit time

(Eq. 5) $$\tau = nL / c,$$

where L is the length of the coil and n is the index of refraction of the optical fiber.

Fig. 2 is a block diagram of an embodiment of an exemplary feedforward component for a closed-loop transfer function 100 according to Fig. 1. As shown in Fig. 2, feed forward component 200 may include disturbances at summing points 202, 204, 206, and 208 based on an optical power noise I_n 205, a shot noise i_s 210, a preamplifier current noise i_n 220, a preamplifier thermal noise i_R 230, a preamplifier voltage noise i_v 240, and an ADC quantization noise n_{ADC} 250. As described herein, a transfer function 100 having a feedforward component 200 may be used to compute a coefficient of random walk (CRW) and a power spectral density (PSD) of noise of FOG 10.

A PSD of shot noise i_s 210 may be represented based on a photodetector current i_D. In one embodiment, a PSD of shot noise i_s 210 may be represented based on the product

(Eq. 6) $$2qi_D=2qI_oR_D\,(1+\cos(\phi_b))\,,$$

where 1^{o}' R_d, and ϕ_b have been previously defined, and q is the charge of the electron.

A PSD of preamplifier thermal noise i_R 230 may be represented based on a temperature T_K of the FOG 10 and a preamplifier impedance R_f. In one embodiment, a PSD of thermal noise i_R 230 may be represented based on the product

(Eq. 7) $$4\,kT_K/R_f,$$

where k is Boltzmann's constant.

A PSD of preamplifier voltage noise i_V 240 may be represented based on a preamplifier voltage e_n, a preamplifier noise gain G_n, and a preamplifier impedance R_f.. In one embodiment, a PSD of preamplifier voltage noise i_v 240 may be

represented based on the product

(Eq. 8) $(e_n G_n / R_f)^2$.

A PSD of ADC quantization noise n_{ADC} 250 may be represented based on an ADC sample period t, a preamplifier impedance R_f a filter gain G_f, and a number of bits b in ADC 137. In one embodiment, a PSD of ADC quantization noise n_{ADC} 250 may be represented based on the product

(Eq. 9) $2t/[12(R_f G_f 2^{b-1})^2]$.

PSDs of optical power noise I_n 205 and preamplifier current noise i_n 220 may be represented based on schemes familiar to those of ordinary skill in the art.

Generally, transfer function 100 may be manipulated using well known control system transform theory to determine relationships between FOG performance parameters and physical parameter(s) associated with FOG component(s). Appendices 1-5 include features related to manipulation of transfer function 100. Relationships for an operating frequency, a bandwidth, a PSD of noise, and a CRW are provided immediately below. As shown, these relationships may depend on FOG physical parameter(s) including at least one of an optical power I_o of light transmitted by a light source 22, an operating phase bias ϕ_b, a photodetector scale factor R_d, a preamplifier impedance R_f, a preamplifier gain G_e, a filter gain G_f, an ADC gain 2^{b-1}, a phase modulator scale factor K_{pm}, and a transit time τ.

Based on a transfer function 100 having a feedforward component 130, an operating frequency ω_o for a FOG 10 may be expressed as

(Eq. 10) $\omega_o = I_o \cdot \sin(\phi_b) \cdot R_d \cdot R_f \cdot G_e \cdot G_f \cdot 2^{b-1} \cdot 1/\tau \cdot G_D \cdot 2^{2-d'} \cdot K_{pm} \cdot$

Based on a transfer function 100 having a feedforward component 130, a

90° bandwidth BW90 for a FOG 10 may be expressed as

(Eq. 11) $$BW90 = (180/\pi \cdot \arg(H(e^{i \cdot w \cdot t}, I_o)) + 90)^{1/2},$$

where $H(z, I_o)$ is defined by the equation

(Eq. 12) $$H(z, I_0) = \omega_0 \cdot \tau \cdot \frac{z^{-(N+1)}}{1 - z^{-1} + \omega_0 \cdot \tau \cdot z^{-(N+M+1)}} \cdot z^{-2},$$

in which N and M are described in Appendices 1-5, as those of ordinary skill in the art will understand.

Based on a transfer function 100 having a feedforward component 130, a 3 dB bandwidth BW3 for a FOG 10 may be expressed as

(Eq. 13) $$BW3 = \left(\left| H\left(e^{i\omega\tau}, I_0\right) \right| - \frac{1}{\sqrt{2}} \right)^{\frac{1}{2}}.$$

Based on a transfer function 100 having a feedforward component 230, a PSD of noise for a FOG 10 may be expressed as

(Eq. 14)

$$PSD = \frac{1}{(K_s \cdot K_I \cdot K_D)^2} \cdot \left[2 \cdot q \cdot I_D + \left(\frac{4 \cdot k \cdot T_K}{R_f} \right) + i_n^2 + \frac{G_n^2 \cdot e_n^2}{R_f^2} + \frac{1}{\left(R_f \cdot G_f \cdot 2^{b-1}\right)^2} \cdot \frac{2 \cdot t}{12} \right].$$

Based on a transfer function 100 having a feedforward component 230, a CRW for a FOG 10 may be expressed as

(Eq. 15) $$CRW = 60 \cdot \frac{180}{\pi} \cdot \sqrt{\frac{PSD}{2}}.$$

Based on the relationships provided in Eqs. 10-15, performance parameters for a FOG 10 may be computed. Generally, a performance parameter may be computed by substituting values of physical parameter(s) in the corresponding relationship for the performance parameter. For example, an operating frequency of a pre-existing FOG may be computed by substituting the values of the physical

parameters of the FOG in the relationship for the operating frequency provided herein. As previously indicated, physical parameters can include, for example, at least one of an optical power I_o of light transmitted by a light source 22, an operating phase bias ϕ_b, a photodetector scale factor R_d, a preamplifier impedance R_f, a preamplifer gain G_e, a filter gain G_f, an ADC gain 2^{b-1}, a digital truncation gain G_D, a DAC gain $2^{2-d'}$, a phase modulator scale factor K_{pm}, and a transit time τ.

The relationships provided in Eqs. 10-15 may be used to design a FOG having desired performance parameter value(s). In one embodiment, performance parameter value(s) may be provided. Based on the relationship(s) corresponding to the performance parameter(s), value(s) associated with physical parameter(s) may be determined for which the computed performance parameter(s) will have or approach the performance parameter value(s). Initial value(s) associated with physical parameter(s) may also be provided. The performance parameter(s) may be computed based on the corresponding relationship(s) and the initial value(s). If a difference can be determined between the computed performance parameter(s) and the performance parameter value(s), then value(s) associated with physical parameter(s) may be iteratively adjusted, and the performance parameter(s) may be iteratively computed based on the iteratively adjusted value(s). For example, a desired value of an operating frequency may be provided, and values of physical parameter(s) may be determined for which a FOG will have the operating frequency value. Also for example, desired values of an operating frequency and a PSD of noise may be provided, and value(s) of physical parameters may be determined for which the operating frequency and the PSD of noise approach the desired values. Generally, the relationships provided in Eqs. 10-15 may be used

with regression schemes familiar to those of ordinary skill in the art.

The schemes described herein are not limited to a particular hardware or software configuration; they may find applicability in many computing or processing environments. The schemes can be implemented in hardware or software, or in a combination of hardware and software. The schemes can be implemented in one or

more computer programs, in which a computer program can be understood to include one or more processor-executable instructions. The computer program(s) can execute on one or more programmable processors, and can be stored on one or more storage media readable by the processor, including volatile and nonvolatile memory and/or storage elements. The programmable processor(s) can access one or more input devices to obtain input data and one or more output devices to communicate output data.

The computer program(s) can be implemented in high level procedural or object oriented programming language to communicate with a computer system. The computer program(s) can also be implemented in assembly or machine language. The language can be compiled or interpreted.

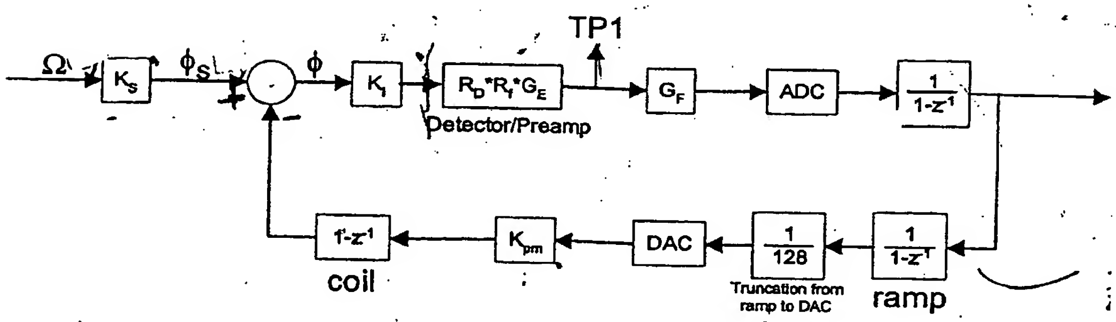
The computer program(s) can be stored on a storage medium or a device (e.g., compact disk (CD), digital video disk (DVD), magnetic disk, internal hard drive, external hard drive, random access memory (RAM), redundant array of independent disks (RAID), or memory stick) that is readable by a general or special purpose programmable computer for configuring and operating the computer when the storage medium or device is read by the computer to perform the schemes described herein.

While the schemes described herein have been particularly shown and described with reference to certain exemplary embodiments, those of ordinary skill in the art will understand that various changes may be made in the form and details of the schemes described herein without departing from the spirit and scope of the present disclosure.

For example, transfer function 100 may be modified based on schemes described herein to compute performance parameters of FOGs including components and/or arrangements of components similar to or different than those of FOG 10 shown in Fig. 3.

Those of ordinary skill in the art will recognize or be able to ascertain many equivalents to the exemplary embodiments described herein by using no more than routine experimentation. Such equivalents are intended to be encompassed by the scope of the present disclosure. Accordingly, the present disclosure is not to be limited to the embodiments described herein and can include practices other than those described, and is to be interpreted as broadly as allowed under prevailing law.

FOG TRANSFER FUNCTION MODEL



$c := 3 \cdot 10^{8}$ $n := 1.45$

Coil and wavelength (meters): $L := 800$ $D := \frac{59.2}{1000}$ $\lambda := 845 \cdot 10^{-9}$ Sagnac scale factor: $K_S := \frac{2 \cdot \pi \cdot L \cdot D}{\lambda \cdot c}$

Modulated detector power (W) and phase bias: $I_0 := 6 \cdot 10^{-6}$ W $\phi_b := \frac{\pi}{2}$ $K_I := I_0 \cdot \sin(\phi_b)$

Detector gain: $R_D := .55$ $\frac{A}{W}$ $R_f := 30 \cdot 10^{3}\ \Omega$ $G_E := 49$ $\frac{V}{V}$

Filt r gain: $G_F := 3.6$ $\frac{V}{V}$

Transit time: $\tau := \frac{n \cdot L}{c}$ $\tau = 2.9 \times 10^{-6}$ $\tau^{-1} = 3.448 \times 10^{5}$

Modulation period: $T := 2 \cdot \tau$ $T = 5.8 \times 10^{-6}$ $T^{-1} = 1.724 \times 10^{5}$

A/D bits: $b_{adc} := 8$ gain $ADC := \frac{2^{b_{adc}}}{2}$ $\frac{lsb}{V}$ (2v range)

Number of bits in 1st integrator $b_1 := 19$

DAC bits: $b_{DAC} := 12$ gain $DAC := \frac{4}{2^{b_{DAC}}}$ V/lsb (4v range)

Digital truncation gain: $G_D := \frac{1}{2^{b_1 - b_{DAC}}}$ $\frac{1}{G_D} = 128$

Phase modulator gain: $K_{pm} := \frac{2 \cdot \pi}{4}$ Rad/V

Closed loop bandwidth:

$\omega_0(I_0) := I_0 \cdot \sin(\phi_b) \cdot R_D \cdot R_f \cdot G_E \cdot G_F \cdot ADC \cdot \frac{1}{\tau} \cdot G_D \cdot DAC \cdot K_{pm}$ $\frac{1}{2 \cdot \pi} \cdot \omega_0(I_0) = 1470.2$ Hz

Computation of 3 dB and 90^0 Bandwidths from Digital Model

Foward and feedback delays, in units of τ: $N := 2.35$ $M := 1$

$$H(z, I_0) := \omega_0(I_0)\cdot\tau\cdot\frac{z^{-(N+1)}}{1 - z^{-1} + \omega_0(I_0)\cdot\tau\cdot z^{-(N+M+1)}}\cdot z^{-2}$$

(The extra z^{-2} factor is to account for output delays through ramp and difference circuit)

$\omega := \omega_0(I_0)$

$$BW3(I_0) := \mathrm{root}\left(\left|H\left(e^{i\cdot\omega\cdot\tau}, I_0\right)\right| - \frac{1}{\sqrt{2}}, \omega\right)$$

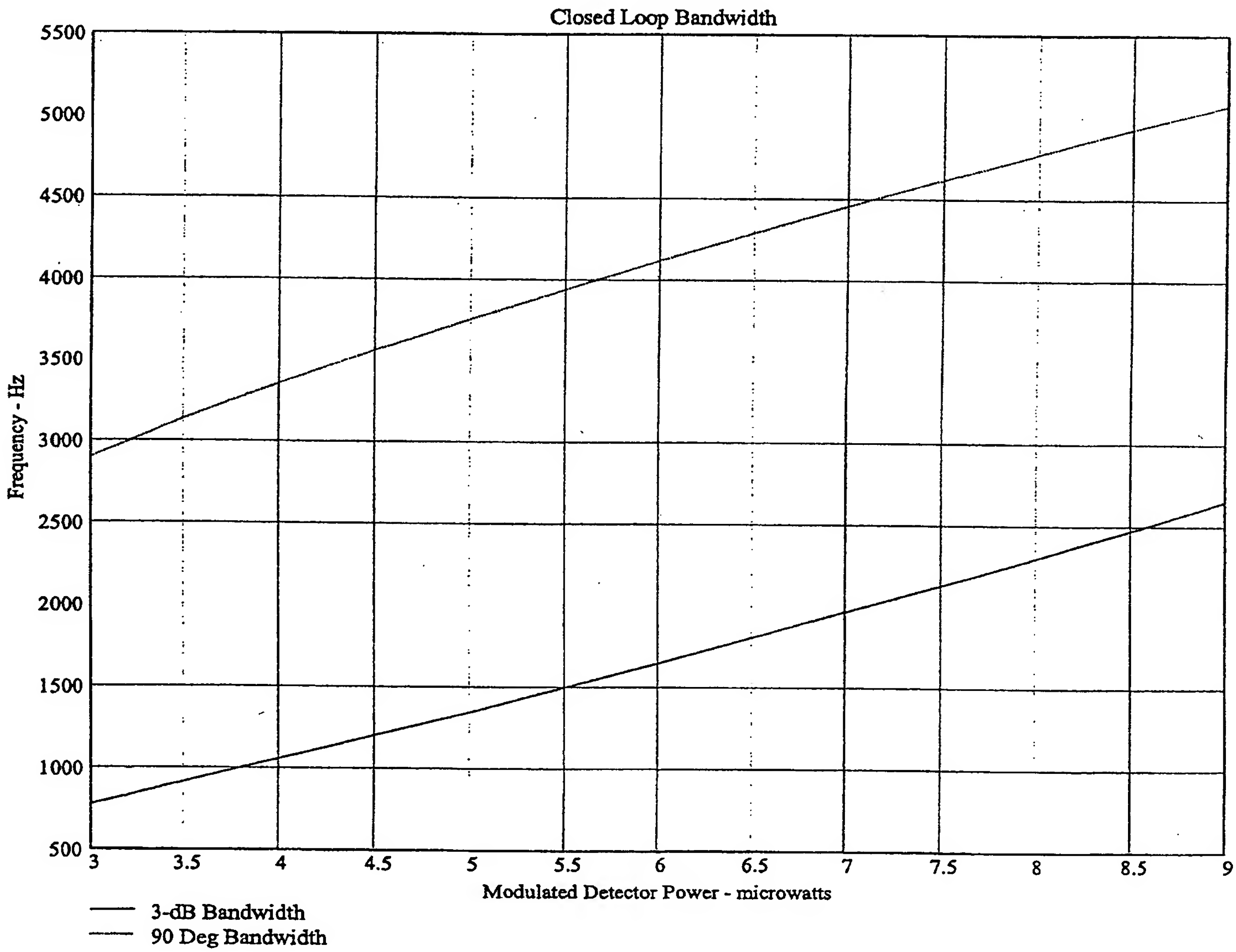
$$\frac{1}{2\cdot\pi}\cdot BW3(I_0) = 1645.8 \text{ Hz}$$

$$BW90(I_0) := \mathrm{root}\left(\frac{180}{\pi}\cdot\arg\left(H\left(e^{i\cdot\omega\cdot\tau}, I_0\right)\right) + 90, \omega\right)$$

$np := 14$ $i := 0 .. np - 1$ $I0_i := \left(\frac{i}{2} + 3\right)\cdot 10^{-6}$

$$Mg(\omega, I_0) := 20\cdot\log\left(\left|H\left(e^{i\cdot\omega\cdot\tau}, I_0\right)\right|\right)$$

$$Ph(\omega, I_0) := \frac{180}{\pi}\cdot\arg\left(H\left(e^{i\cdot\omega\cdot\tau}, I_0\right)\right)$$



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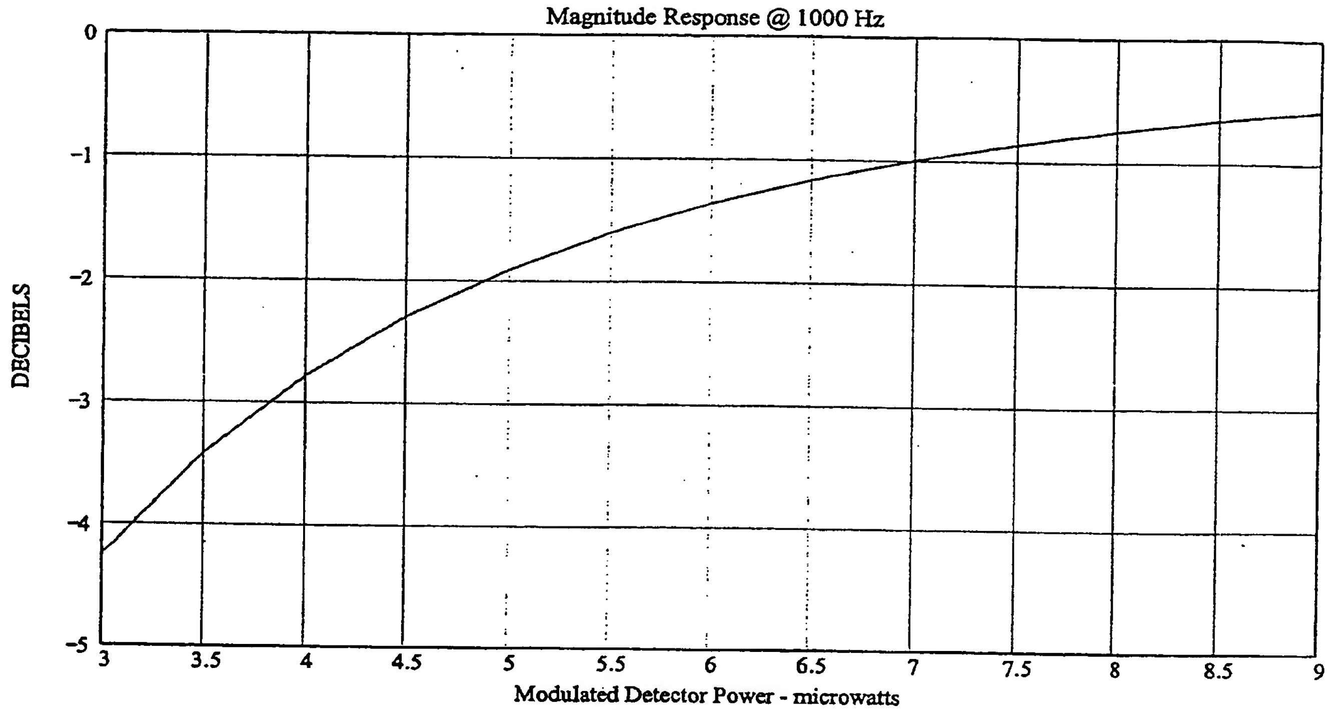
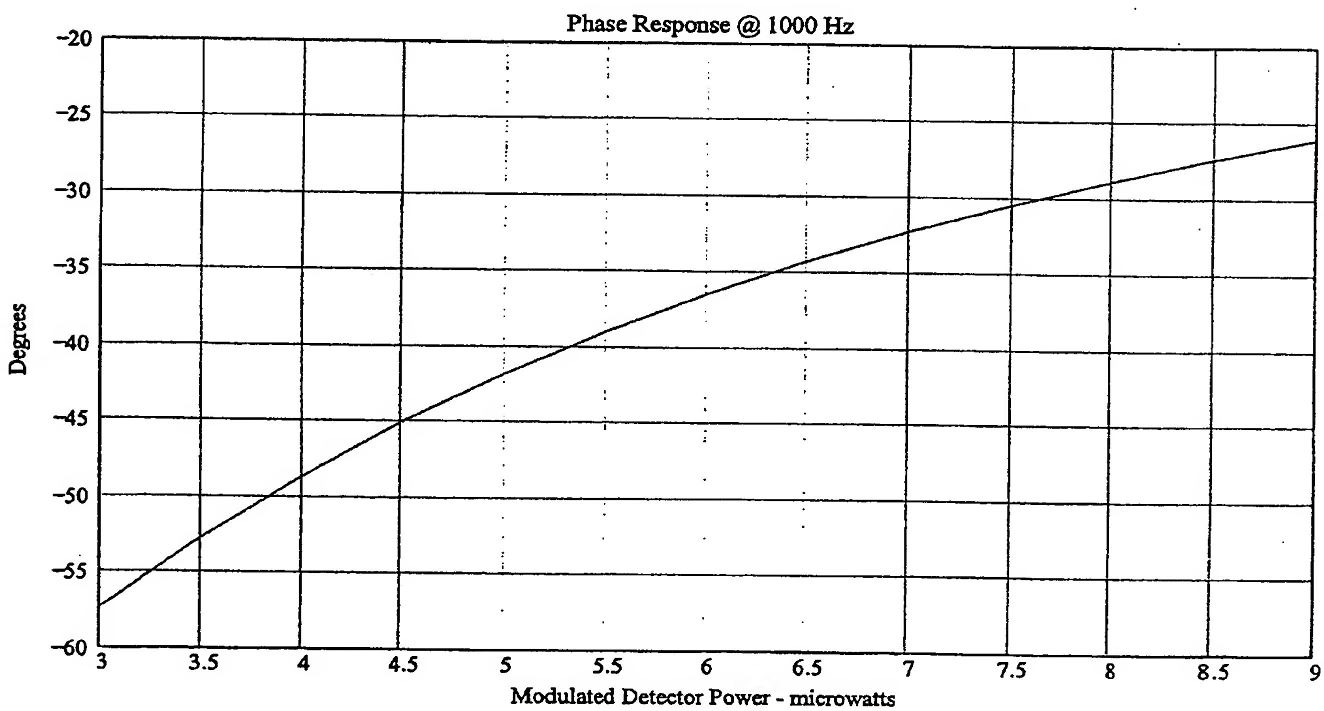
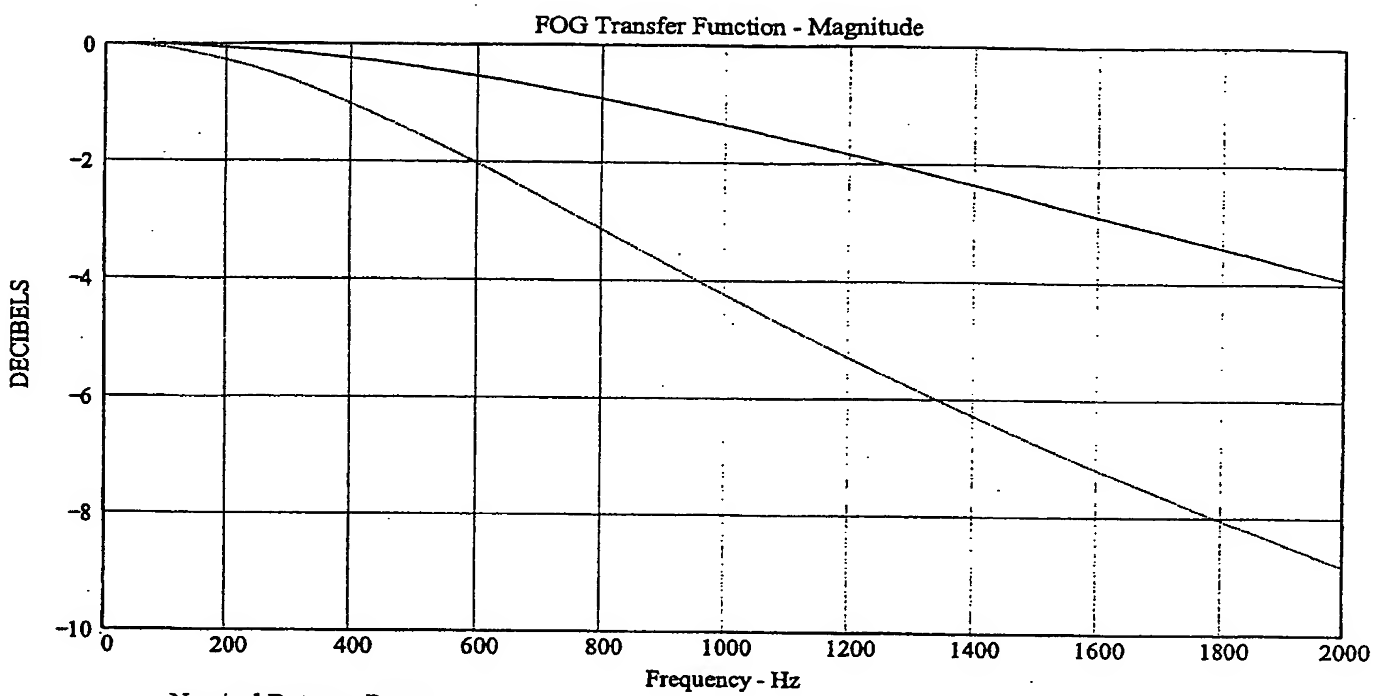
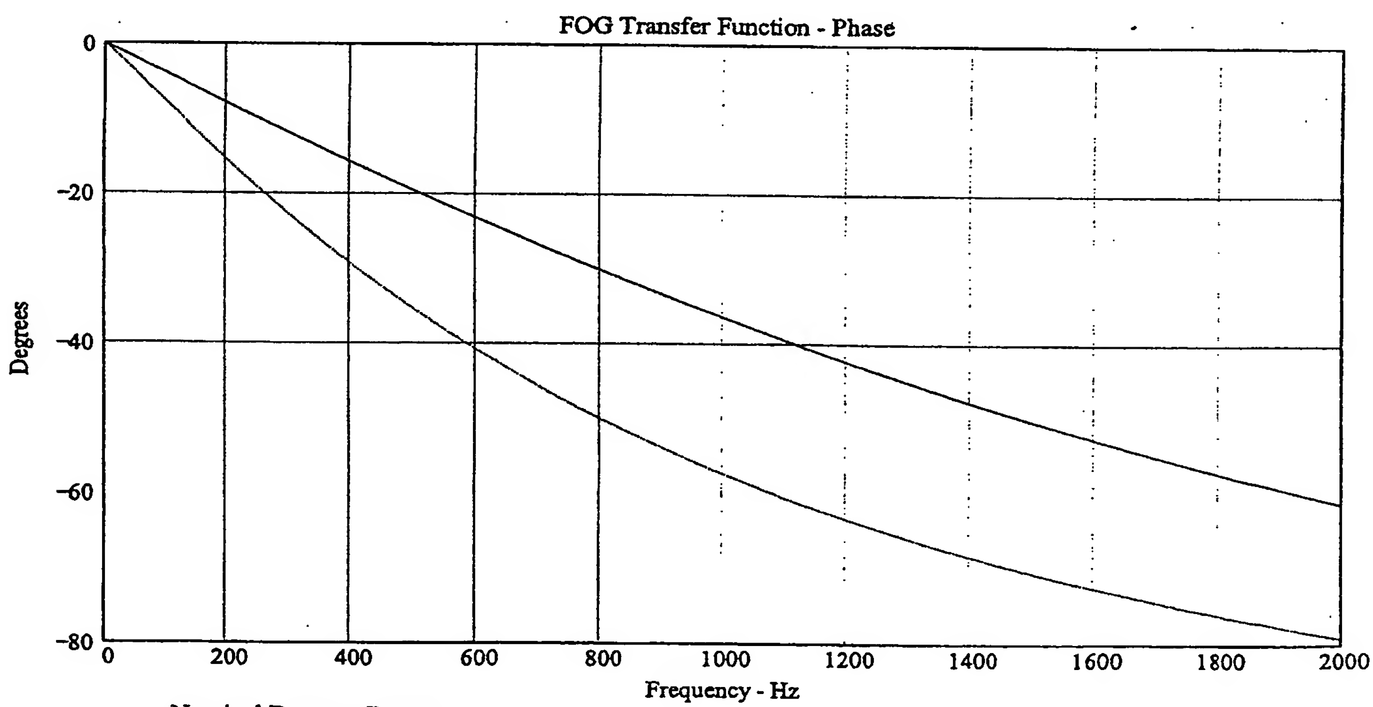

Magnitude Response @ 1000 Hz
0
-1
-2
-3
-4
-5
DECIBELS
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
Modulated Detector Power - microwatts


Phase Response @ 1000 Hz
-20
-25
-30
-35
-40
-45
-50
-55
-60
Degrees
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
Modulated Detector Power - microwatts

f := 0,50..2000



—— Nominal Detector Power
—— Half Detector Power

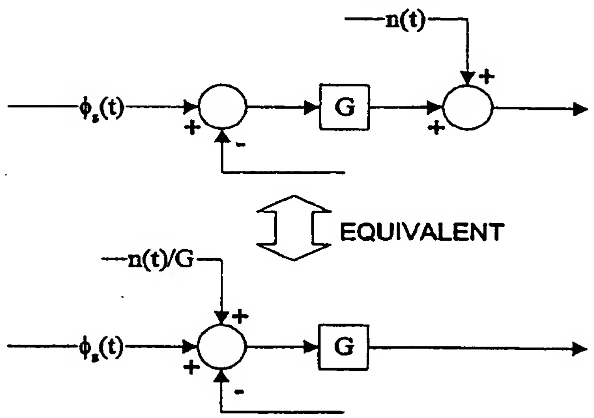


—— Nominal Detector Power
—— Half Detector Power

A STUDY OF GAIN DISTRIBUTION AND RANDOM WALK

The following four rules will be used to study gain distribution:

1. BACKING OUT A NOISE TERM THROUGH A GAIN BLOCK



2. For a modulation/demodulation block containing the modulation function $M(t)$, use:

$$M(t)^2 = 1 \quad \text{or} \quad M(t) = M(t)^{-1}$$

3. If $n(t)$ is a white noise random process, then for the purpose of statictisal computations:

$$n(t) = M(t) \cdot n(t)$$

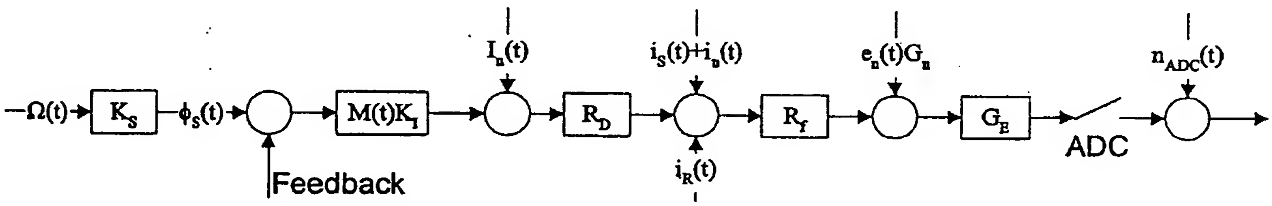
since both terms have the same statistics.

4. If $S_{nn}(\omega)$ is the PSD of a random process $n(t)$, and $M(t)$ is a square wave modulation function of frequency ω_0, then, $y(t) = M(t) \cdot n(t)$ has PSD:

$$S_{yy}(\omega) = \frac{4}{\pi^2} \cdot \sum_n \frac{1}{n^2} \cdot S_{xx}(\omega - \omega_0)$$

$n = \pm 1, \pm 3, \ldots$

Signals, Noise Sources, and Gains



$\Omega(t)$ is the input rate

$K_S = \frac{2 \cdot \pi \cdot L \cdot D}{\lambda \cdot c}$ is the Sagnac scale factor

$K_I = I_0 \cdot \sin(\phi_b)$ is the phase gain at the operating bias point, ϕ_b

I_0 is the optical power (1/2 peak)

$I_n(t)$ is the optical power noise

R_D is the detector scale factor in A/W

R_f is the feedback resistor in the transimpedance amplifier

G_E is the net voltage gain from the detector to the A/D input

G_n is the noise gain of the transimpedance amplifier

$i_S(t)$ is the shot current

$i_R(t)$ is the feedback resistor thermal noise

$i_n(t)$ is the amplifier current noise

$e_n(t)$ is the amplifier voltage noise

$n_{adc}(t)$ is the A/D quantization noise

$ADC = 2^{b-1}$ is the gain, in lsb/V, for a A/D with b bits.

Performing this procedure for the Sagnac phase gives

$$\phi_S(t) + \frac{M(t)\cdot I_n(t)}{K_I} + \frac{i_S(t)}{K_I\cdot R_D} + \frac{i_R(t)}{K_I\cdot R_D} + \frac{i_n(t)}{K_I\cdot R_D} + \frac{M(t)\cdot e_n(t)\cdot G_n}{K_I\cdot R_D\cdot R_f} + \frac{n_{adc}(t)}{K_I\cdot R_D\cdot R_f\cdot G_E\cdot 2^{b-1}}$$

A factor $M(t)$ indicates those noise sources that may not be white. Backing out all the way to the input rate gives:

$$\Omega(t) + \frac{M(t)\cdot I_n(t)}{K_S\cdot K_I} + \frac{i_S(t)}{K_S\cdot K_I\cdot R_D} + \frac{i_R(t)}{K_S\cdot K_I\cdot R_D} + \frac{i_n(t)}{K_S\cdot K_I\cdot R_D} + \frac{M(t)\cdot e_n(t)\cdot G_n}{K_S\cdot K_I\cdot R_D\cdot R_f} + \frac{n_{adc}(t)}{K_S\cdot K_I\cdot R_D\cdot R_f\cdot G_E\cdot 2^{b-1}}$$

This expression can be used to compute the <u>net sampled PSD</u> of all noise sources. The PSD of the shot current is:

$P_{iS}(\omega) = 2\cdot q\cdot i_D \ \frac{A^2}{hz}$ where $i_D = I_0\cdot\left(1+\cos\left(\phi_b\right)\right)\cdot R_D$ is the detector current

and $q := 1.602\cdot 10^{-19}$ is the electron charge

The resistor thermal noise has PSD: $P_{iR}(\omega) = \frac{4\cdot k\cdot T_K}{R_f}\ \frac{A^2}{hz}$ where $k := 1.380658\cdot 10^{-23}$ is Boltzman's constant.

$T_K := 298$ is the Kelvin temperature is

The A/D quantization noise has PSD: $P_{acd}(\omega) = \frac{2\cdot\tau}{12}$ where τ is the sample period.

With the following parameter values:

$R_f := 30\cdot 10^3$ $R_D := .55$ $b := 8$ $I_0 := 6\cdot 10^{-6}$ $G_E := 49\cdot 3.6$ $G_E = 176.4$

$i_n := 0\cdot 10^{-1}\frac{A}{\sqrt{hz}}$ $e_n := 32\cdot 10^{-}\frac{V}{\sqrt{hz}}$ $G_n := 1$ $c := 3\cdot 10^8$ $n := 1.45$

<u>Coil and wavelength (meters):</u> $L := 600$ $D := \frac{59.2}{1000}$ $\lambda := 845\cdot 10^{-9}$ $K_S := \frac{2\cdot\pi\cdot L\cdot D}{\lambda\cdot c}$

<u>Transit time:</u> $\tau := \frac{n\cdot L}{c}$ $\tau = 2.9\times 10^{-6}$ $\tau^{-1} = 3.448\times 10^5$

<u>Modulation period:</u> $T := 2\cdot\tau$ $T = 5.8\times 10^{-6}$ $T^{-1} = 1.724\times 10^5$

The net PSD, with units of $\left(\frac{rad}{sec}\right)^2\cdot\frac{1}{hz}$ can be written:

$$P_0(\phi_b) := \frac{1}{\left(K_S\cdot I_0\cdot\sin\left(\phi_b\right)\cdot R_D\right)^2}\cdot\left[2\cdot q\cdot I_0\cdot\left(1+\cos\left(\phi_b\right)\right)\cdot R_D + \left(\frac{4\cdot k\cdot T_K}{R_f}\right) + i_n^2 + \frac{G_n^2\cdot e_n^2}{R_f^2} + \frac{1}{\left(R_f\cdot G_E\cdot 2^{b-1}\right)^2}\cdot\frac{2\cdot\tau}{12}\right]$$

and from the PSD, the random walk coefficient is: $\sigma_{rw}(\phi_b) := 60\cdot\frac{180}{\pi}\cdot\sqrt{\frac{P_0(\phi_b)}{2}}$ $\frac{deg}{\sqrt{hr}}$

$i_D = I_0 \cdot (1+\cos(\phi_b)) \cdot R_D \qquad i_S = \sqrt{2 \cdot q \cdot i_D}$

$K_I = I_0 \cdot \sin(\phi_b)$

$$\left[\left(\phi_S(t) \cdot M(t) \cdot K_I \cdot R_D + i_S(t) + i_R(t) + i_n(t)\right) \cdot R_f + e_n(t) \cdot G_n\right] \cdot G_E \cdot 2^{b-1} + n_{adc}(t)$$

$M(t)^2 = 1$

$$M(t) \cdot K_I \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1} \cdot \left[\phi_S(t) + \frac{M(t) \cdot (i_S(t) + i_R(t) + i_n(t))}{K_I \cdot R_D} + \frac{M(t) \cdot e_n(t) \cdot G_n}{K_I \cdot R_D \cdot R_f} + \frac{M(t) \cdot n_{adc}(t)}{K_I \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1}}\right]$$

But $\quad M(t) \cdot i_S(t) = i_S(t) \qquad M(t) \cdot i_R(t) = i_R(t) \qquad M(t) \cdot i_n(t) = i_n(t)$

$M(t) \cdot e_n(t) = e_n(t) \qquad M(t) \cdot n_{adc}(t) = n_{adc}(t)$

$$M(t) \cdot K_I \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1} \cdot \left(\phi_S(t) + \frac{i_S(t) + i_R(t) + i_n(t)}{K_I \cdot R_D} + \frac{e_n(t) \cdot G_n}{K_I \cdot R_D \cdot R_f} + \frac{n_{adc}(t)}{K_I \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1}}\right)$$

$$\phi_S(t) + \frac{i_S(t)}{K_I \cdot R_D} + \frac{i_R(t)}{K_I \cdot R_D} + \left(\frac{i_n(t)}{K_I \cdot R_D} + \frac{e_n(t) \cdot G_n}{K_I \cdot R_D \cdot R_f}\right) + \frac{n_{adc}(t)}{K_I \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1}}$$

$\phi_S(t) = K_S \cdot \Omega(t) \qquad K_S = \dfrac{2 \cdot \pi \cdot L \cdot D}{\lambda \cdot c}$

$$K_S \cdot \Omega(t) + \frac{i_S(t)}{K_I \cdot R_D} + \frac{i_R(t)}{K_I \cdot R_D} + \left(\frac{i_n(t)}{K_I \cdot R_D} + \frac{e_n(t) \cdot G_n}{K_I \cdot R_D \cdot R_f}\right) + \frac{n_{adc}(t)}{K_I \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1}}$$

$$K_S \cdot \left[\Omega(t) + \frac{i_S(t)}{K_S \cdot K_I \cdot R_D} + \frac{i_R(t)}{K_S \cdot K_I \cdot R_D} + \left(\frac{i_n(t)}{K_S \cdot K_I \cdot R_D} + \frac{e_n(t) \cdot G_n}{K_S \cdot K_I \cdot R_D \cdot R_f}\right) + \frac{n_{adc}(t)}{K_S \cdot K_I \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1}}\right]$$

$P_{iS}(\omega) = 2 \cdot q \cdot i_D \quad \dfrac{A^2}{hz} \qquad P_{iR}(\omega) = \dfrac{4 \cdot k \cdot T_K}{R_f} \dfrac{A^2}{hz} \qquad i_D = I_0 \cdot (1+\cos(\phi_b)) \cdot R_D$

$c_{rw} = 60 \cdot \dfrac{180}{\pi} \cdot \sqrt{\dfrac{P_0}{2}} \qquad ((P_0)) = \left(\dfrac{rad}{sec}\right)^2 \cdot \dfrac{1}{hz} \qquad ((c_{rw})) = \dfrac{deg}{\sqrt{hz}}$

$$P_0 = \frac{2\cdot q\cdot i_D}{(K_S\cdot K_I\cdot R_D)^2} + \frac{1}{(K_S\cdot K_I\cdot R_D)^2}\cdot\left(\frac{4\cdot k\cdot T_K}{R_f}\right) + \frac{{i_n}^2}{(K_S\cdot K_I\cdot R_D)^2} + \frac{{G_n}^2\cdot{e_n}^2}{(K_S\cdot K_I\cdot R_D\cdot R_f)^2} + \frac{1}{\left(K_S\cdot K_I\cdot R_D\cdot R_f\cdot G_E\cdot 2^{b-1}\right)^2}\cdot\frac{2\cdot\tau}{12}$$

$$K_I = I_0\cdot\sin(\phi_b) \qquad i_D = I_0\cdot(1+\cos(\phi_b))$$

$$P_0 = \frac{1}{(K_S\cdot K_I\cdot R_D)^2}\cdot\left[2\cdot q\cdot i_D + \left(\frac{4\cdot k\cdot T_K}{R_f}\right) + {i_n}^2 + \frac{{G_n}^2\cdot{e_n}^2}{{R_f}^2} + \frac{1}{\left(R_f\cdot G_E\cdot 2^{b-1}\right)^2}\cdot\frac{2\cdot\tau}{12}\right]$$

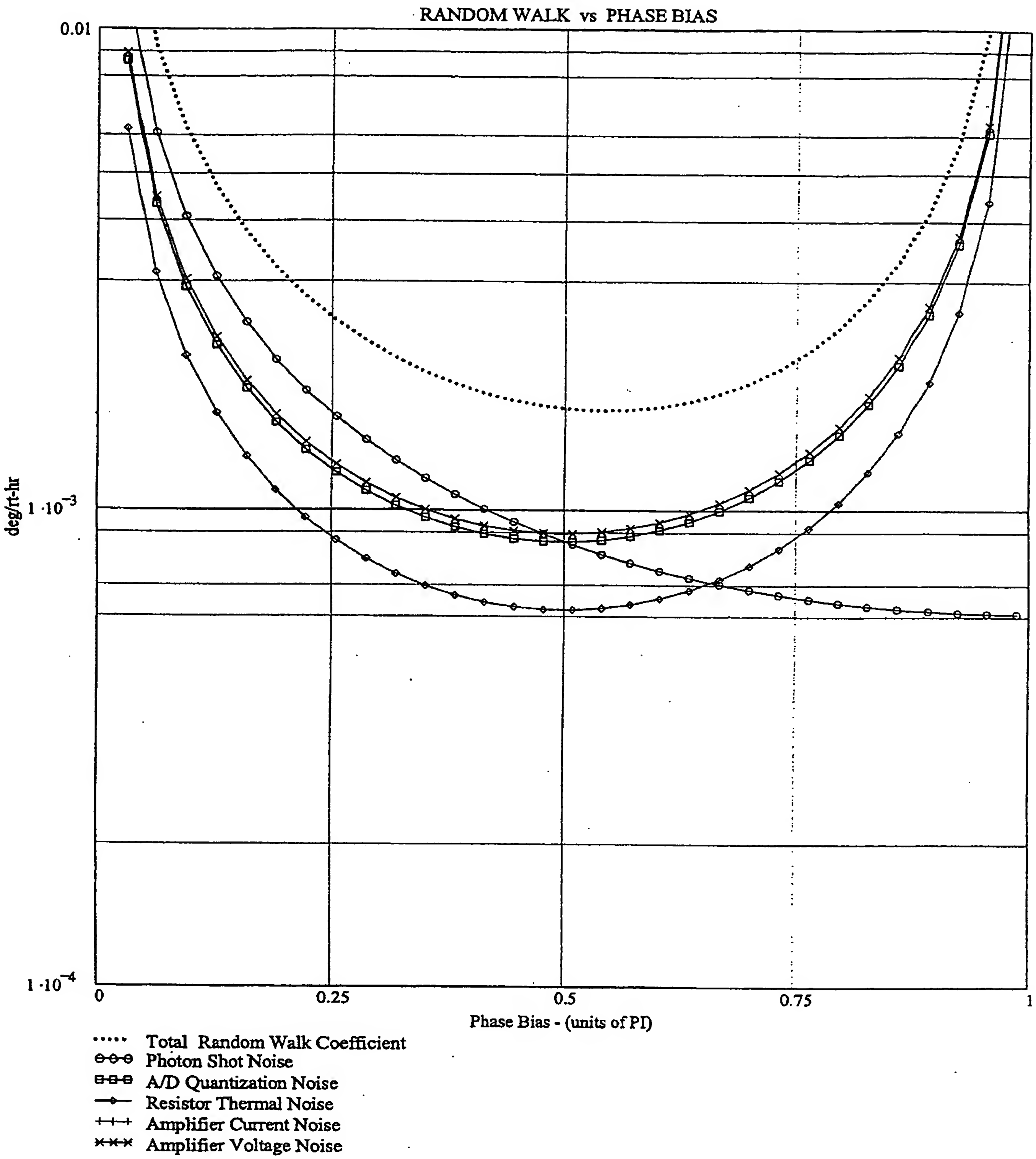
$$P_0 = \frac{1}{(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D)^2}\cdot\left[2\cdot q\cdot I_0\cdot(1+\cos(\phi_b))\cdot R_D + \left(\frac{4\cdot k\cdot T_K}{R_f}\right) + {i_n}^2 + \frac{{G_n}^2\cdot{e_n}^2}{{R_f}^2} + \frac{1}{\left(R_f\cdot G_E\cdot 2^{b-1}\right)^2}\cdot\frac{2\cdot\tau}{12}\right]$$

$$\phi_b := 0, .1 .. \pi$$

$$\mathrm{shott}(\phi_b) := 60\cdot\frac{180}{\pi}\cdot\sqrt{\frac{2\cdot q\cdot I_0\cdot(1+\cos(\phi_b))\cdot R_D}{2\cdot(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D)^2}} \qquad \mathrm{Rtherm}(\phi_b) := 60\cdot\frac{180}{\pi}\cdot\sqrt{\frac{1}{2\cdot(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D)^2}\cdot\left(\frac{4\cdot k\cdot T_K}{R_f}\right)}$$

$$\mathrm{amp\_i}(\phi_b) := 60\cdot\frac{180}{\pi}\cdot\sqrt{\frac{{i_n}^2}{2\cdot(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D)^2}} \qquad \mathrm{amp\_v}(\phi_b) := 60\cdot\frac{180}{\pi}\cdot\sqrt{\frac{1}{2\cdot(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D)^2}\cdot\frac{{G_n}^2\cdot{e_n}^2}{{R_f}^2}}$$

$$\mathrm{adc}(\phi_b) := 60\cdot\frac{180}{\pi}\cdot\sqrt{\frac{1}{2\cdot(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D)^2}\cdot\frac{1}{\left(R_f\cdot G_E\cdot 2^{b-1}\right)^2}\cdot\frac{2\cdot\tau}{12}}$$



$$40\cdot10^{-6}\cdot\frac{\pi}{2}\cdot\frac{1}{K_S}\cdot\frac{180}{\pi}\cdot3600\cdot10^{-3} = 0.015\frac{deg}{hr}\cdot\frac{1}{\mu A}$$

Point Sensitivities for Nonrandom Inputs

$$\phi_b := \frac{\pi}{2}$$

$$\frac{10^{-9}}{K_S\cdot(I_0\cdot\sin(\phi_b))}\cdot\frac{180}{\pi}\cdot 3600 = 39.048 \qquad \frac{deg}{hr}\cdot\frac{1}{\eta W}$$ 1N @ Detector input

$$\frac{10^{-9}}{K_S\cdot(I_0\cdot\sin(\phi_b))\cdot R_D}\cdot\frac{180}{\pi}\cdot 3600 = 70.996 \qquad \frac{deg}{hr}\cdot\frac{1}{\eta A}$$ 1N @ Detector output

$$\frac{10^{-6}}{K_S\cdot(I_0\cdot\sin(\phi_b))\cdot R_D\cdot R_f}\cdot\frac{180}{\pi}\cdot 3600 = 2.367 \qquad \frac{deg}{hr}\cdot\frac{1}{\mu V}$$ 1N @ Trans-impedence Amp. output

$$\frac{10^{-6}}{K_S\cdot(I_0\cdot\sin(\phi_b))\cdot R_D\cdot R_f\cdot G_E}\cdot\frac{180}{\pi}\cdot 3600 = 0.013 \qquad \frac{deg}{hr}\cdot\frac{1}{\mu V}$$ 1N @ A/D input

$V_0 := 10^{-6}$ $\quad n := 2$ $\quad PPM := 100\cdot 10^{-6}$ (tuning error) $\quad K_{pm} := \frac{2\cdot\pi}{4}$

$$\pi^2\cdot\frac{K_{pm}}{K_S}\cdot PPM\cdot\frac{n^2}{n^2-1}\cdot V_0\cdot\left(\frac{180}{\pi}\cdot 3600\right) = 0.000484 \qquad \frac{deg}{hr\cdot\mu V}$$ 2N @ IOC input

Hervé Lefèvre's A/D bits criterion:

$\omega_1 := 2\cdot\pi\cdot 400\cdot 10^3$ $\quad \omega_2 := 2\cdot\pi\cdot 800\cdot 10^3$ $\quad B_L := \frac{\omega_1\cdot\omega_2}{4\cdot(\omega_1+\omega_2)}$ $\quad \frac{B_L}{1000} = 418.9$ KHz $\quad \log_2(x) := \frac{\log(x)}{\log(2)}$

Criterion is rms-noise = lsb $\quad \sqrt{i_S^2\cdot B_L}\cdot R_f\cdot G_E = \frac{1}{2^{b-1}}$ (based on shot noise only, since it should dominate)

or $\quad b := 1 + \log_2\left[\frac{1}{R_f\cdot G_E\cdot\sqrt{B_L\cdot 2\cdot q\cdot I_0\cdot(1+\cos(\phi_b))\cdot R_D}}\right]$ $\quad b = 9.1$ bits

$b_{ramp} := 19$ $\quad b_{adc} := 8$ $\quad \frac{\pi}{2^{b_{ramp}-1}}\cdot\frac{1}{K_S}\cdot\frac{\left(2^{b_{adc}-1}-1\right)}{\tau}\cdot\frac{180}{\pi} = 34155.7 \quad \frac{deg}{sec^2}$ (max angular acceleration)

$b_{out} := 6$ $\quad \frac{\pi}{2^{b_{out}-1}}\cdot\frac{1}{K_S}\cdot\tau\cdot\frac{180}{\pi}\cdot 3600 = 0.066703$ arcsec (LSB value for b_{out} output bits from ramp)

$R_f \cdot G_E = 5.292 \times 10^6 \quad I_0 = 6 \times 10^{-6} \quad \phi_b = 0.5\pi \quad K_S = 0.88 \quad \tau = 2.9 \times 10^{-6} \quad DAC := \frac{4}{2^{12}} \quad K_{dig} := \frac{1}{128}$

One lsb, at A/D $\quad b := 8$ $\qquad$ at 1st integrator

$$\frac{1}{K_S \cdot I_0 \cdot \sin(\phi_b) \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1}} \cdot \frac{180}{\pi} \cdot 3600 = 104.811 \quad \frac{\left(\frac{deg}{hr}\right)}{bit}$$

$$\frac{K_{pm} \cdot DAC \cdot K_{dig}}{K_S} \cdot 3600 \cdot \frac{180}{\pi} = 2.808 \quad \frac{\left(\frac{deg}{hr}\right)}{bit}$$

$$K_S \cdot I_0 \cdot \sin(\phi_b) \cdot R_D \cdot R_f \cdot G_E \cdot 2^{(b-1)} \cdot \frac{\pi}{180 \cdot 3600} = 9.541 \times 10^{-3} \quad \frac{it}{\left(\frac{:g}{hr}\right)}$$

$$\frac{10^6}{I_0 \cdot \sin(\phi_b) \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1}} = 447.359 \quad \frac{\mu Rad}{bit}$$

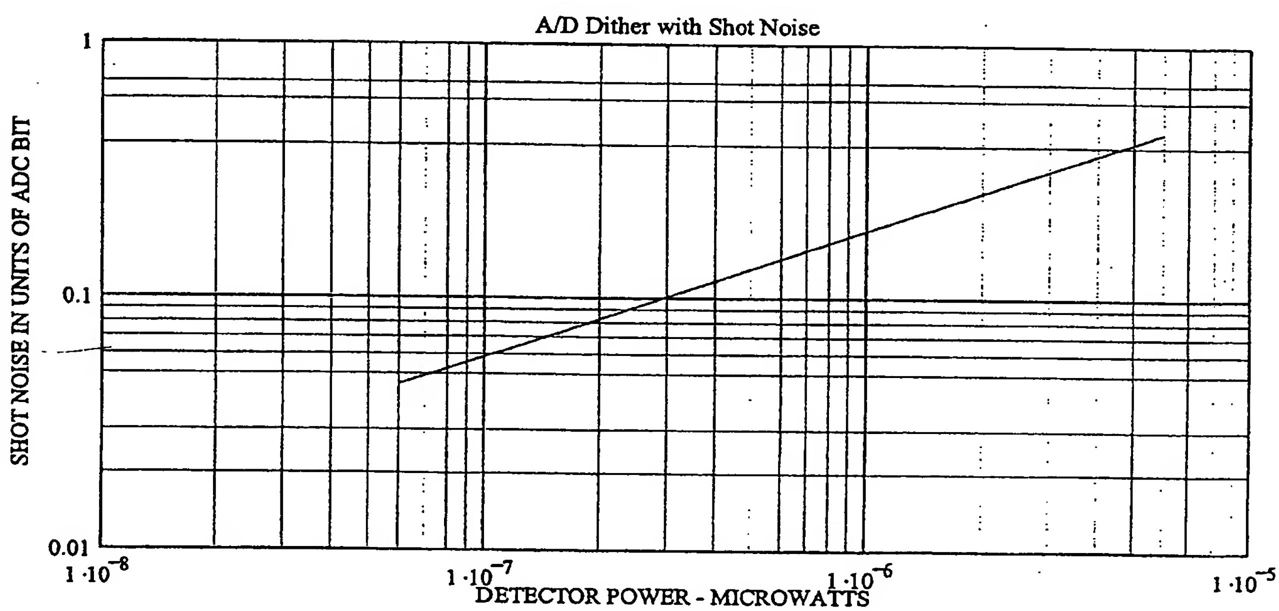
$$\frac{I_0 \cdot \sin(\phi_b) \cdot R_D \cdot R_f \cdot G_E \cdot 2^{b-1}}{10^6} = 2.235 \times 10^{-3} \quad \frac{bit}{\mu Rad}$$

$$\frac{10^6}{R_D \cdot R_f \cdot G_E \cdot 2^{b-1}} = 2.684 \times 10^{-3} \quad \frac{\mu Watt}{bit}$$

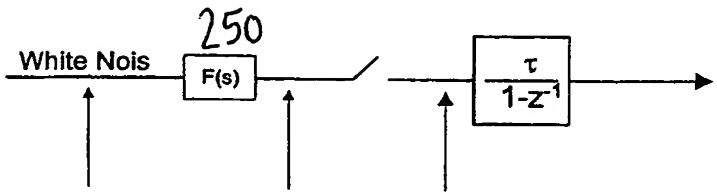
$$\frac{R_D \cdot R_f \cdot G_E \cdot 2^{b-1}}{10^6} = 372.557 \quad \frac{bit}{\mu Watt}$$

$$shot(I_0) := R_f \cdot G_E \cdot \sqrt{B_L \cdot 2 \cdot q \cdot I_0 \cdot (1 + \cos(\phi_b)) \cdot R_D}$$

$N := 100 \qquad i := 1..N \qquad I0_i := I_0 \cdot \frac{i}{N} \qquad Vrms\_shot_i := shot(I0_i) \qquad Vbit := \frac{1}{2^{b-1}}$



FOG ANGLE RANDOM WALK



$$\sigma_I = \tau\cdot\sqrt{N\cdot\left(P_0\cdot B_L + \sigma_{quant}^{\ 2}\right)}$$

$N = \dfrac{T_{ave}}{\tau}$ = number of samples in sum.

For $N\cdot\tau = T_{ave} = 1\cdot hr$, this gives the ARW coefficient. (This formula assumes that the individual samples are uncorrellated, which will be the case if the filter time constant is much less than the sample period.)

$P(\omega) = P_0$ (PSD)

$\sigma_f = \sqrt{P_0\cdot B_L}$ $\qquad \sigma_s = \sqrt{P_0\cdot B_L + \sigma_{quant}^{\ 2}}$

B_L= noise equivalent bandwidth of filter $F(s)$

$\sigma_{quant}^{\ 2} = \dfrac{lsb}{12}$ (A/D quantization noise)

Some Noise Equivalent Bandwidths

$F(s)$	B_L (Hz)
$\dfrac{\omega_0}{s+\omega_0}$	$\dfrac{\omega_0}{4}$
$\dfrac{\omega_1}{s+\omega_1}\cdot\dfrac{\omega_2}{s+\omega_2}$	$\dfrac{\omega_1\cdot\omega_2}{4\cdot(\omega_1+\omega_2)}$
$\dfrac{\omega_0^{\ 2}}{s^2+2\cdot\xi\cdot\omega_0\cdot s+\omega_0^{\ 2}}$	$\dfrac{\omega_0}{8\cdot\xi}$
$\dfrac{1-e^{-s\cdot T}}{s\cdot T}$	$\dfrac{1}{2\cdot T}$ (average over period T)

Definition

$$B_L = \int_0^{\infty}\left(\left|F(2\cdot\pi\cdot i\cdot f)\right|\right)^2 df$$

Set param ter values

The PSD of the shot current is:

$$P_{iS}(\omega) = 2\cdot q\cdot i_D \quad \frac{A^2}{hz}$$ wher $i_D = I_0\cdot(1+\cos(\phi_b))\cdot R_D$ is the detector current

and $q := 1.602\cdot 10^{-19}$ is the electron charge

The resistor thermal noise has PSD: $P_{iR}(\omega) = \frac{4\cdot k\cdot T_K}{R_f}\ \frac{A^2}{hz}$ where $k := 1.380658\cdot 10^{-23}$ is Boltzman's constant.

$T_K := 298$ is the Kelvin temperature is $T_K := 0$ (set $T_K = 0$ when thermal noise is included in amplifier output noise spec)

With the following parameter values: $\phi_b := \frac{\pi}{2}$

$R_f := 30\cdot 10^3$ $R_D := .55$ $b := 8$ $I_0 := 6\cdot 10^{-6}$ $G_E := 49$ $G_F := 3.6$

$i_n := 0\cdot 10^{-12}\frac{A}{\sqrt{hz}}$ $e_n := 32\cdot 10^{-9}\frac{V}{\sqrt{hz}}$ $G_n := 1$ $c := 3\cdot 10^8$ $n := 1.45$ (set $i_n = 0$ when current noise is included in amplifier output noise spec)

Coil and wavelength (meters): $L := 600$ $D := \frac{59.2}{1000}$ $\lambda := 845\cdot 10^{-9}$ $K_S := \frac{2\cdot\pi\cdot L\cdot D}{\lambda\cdot c}$

Transit time: $\tau := \frac{n\cdot L}{c}$ $\tau = 2.9\times 10^{-6}$ $\tau^{-1} = 3.448\times 10^5$

Modulation period: $T := 2\cdot\tau$ $T = 5.8\times 10^{-6}$ $T^{-1} = 1.724\times 10^5$ $f_M := \frac{1}{T}$ $f_s := \frac{1}{\tau}$

$\omega_M := 2\cdot\pi\cdot f_M$ $\omega_s := 2\cdot\pi\cdot f_s$

Th excess noise has PSD: $P_{ex}(\omega) = \frac{i_D^2}{\Delta f}\ \frac{A^2}{hz}$ where Δf is the optical spectrum frequency width

In terms of the full width at half maximum, Δf_{fwhm}, Δf is computed as:

$\Delta\lambda_{fwhm} := 18\cdot 10^{-9}$ $\Delta f_{fwhm} := \frac{c\cdot\Delta\lambda_{fwhm}}{\lambda^2}$ $\Delta f := \sqrt{\frac{\pi}{2\cdot\ln(2)}}\cdot\Delta f_{fwhm}$ $\sqrt{\frac{\pi}{2\cdot\ln(2)}} = 1.505$

Low pass filter:

$\omega_1 := 2\cdot\pi\cdot 4.42\cdot 10^6$ $\omega_2 := 2\cdot\pi\cdot 3.62\cdot 10^6$ $F(s) := \frac{\omega_1}{s+\omega_1}\cdot\frac{\omega_2}{s+\omega_2}$ $B_L := \frac{\omega_1\cdot\omega_2}{4\cdot(\omega_1+\omega_2)}$ $B_L = 3.126\times 10^6$ Hz

A/D bits: $b := 8$ gain $ADC := \frac{2^b}{2}\ \frac{lsb}{v}$ (+/- 2v range)

$$\frac{2\cdot\pi}{2^6}\cdot\frac{1}{K_S}\cdot\tau\cdot\frac{180}{\pi}\cdot 3600 = 0.066703 \quad \frac{arcsec}{lsb}$$

Compute ARW

1. The net PSD before the filter $F(s)$, with units of $\frac{volt^2}{hz}$ can be written:

$$PSD_0(I_0, \phi_b) := \left[2 \cdot q \cdot I_0 \cdot (1 + \cos(\phi_b)) \cdot R_D + \frac{[I_0 \cdot (1 + \cos(\phi_b)) \cdot R_D]^2}{\Delta f} + \left(\frac{4 \cdot k \cdot T_K}{R_f}\right) + i_n^2 + \frac{G_n^2 \cdot e_n^2}{R_f^2}\right] \cdot R_f^2 \cdot G_E^2$$

$$P_0 := PSD_0(I_0, \phi_b) \qquad P_0 = 6.81 \times 10^{-12} \quad \frac{volt^2}{hz}$$

2. The rms filtered noise after $F(s)$ is: $\sigma_f := G_F \cdot \sqrt{P_0 \cdot B_L}$ $\quad \sigma_f = 0.0166$ vrms

3. The rms sampled noise is: $\sigma_s := \sqrt{(ADC \cdot \sigma_f)^2 + \frac{1}{12}}$ $\quad \sigma_s = 2.146$ lsb rms

4. Accumulate samples for one hour and multiply by dt to convert to angle:

$$\sigma_I := \frac{180}{\pi} \cdot \frac{1}{(K_S \cdot I_0 \cdot \sin(\phi_b) \cdot R_D \cdot R_f \cdot G_E \cdot G_F \cdot ADC)} \cdot \tau \cdot \sqrt{\frac{3600}{\tau} \cdot \sigma_s^2} \qquad \sigma_I = 0.006383 \quad \frac{deg}{\sqrt{hr}}$$

$b_{adc} := 8 \qquad \Delta Vh := 1$

$$\frac{2^{b_{adc}-1}}{\Delta Vh} \cdot G_F \cdot \sqrt{\left[2 \cdot q \cdot I_0 \cdot (1 + \cos(\phi_b)) \cdot R_D + \frac{[I_0 \cdot (1 + \cos(\phi_b)) \cdot R_D]^2}{\Delta f} + \frac{G_n^2 \cdot e_n^2}{R_f^2}\right] \cdot R_f^2 \cdot G_E^2 \cdot B_L} = 2.126$$

8.933 0.482 6.309 0.871

$$\frac{180}{\pi} \cdot \frac{1}{(K_S \cdot I_0 \cdot \sin(\phi_b) \cdot R_D)} \cdot \sqrt{3600 \cdot \tau} \cdot \sqrt{B_L} \cdot \sqrt{\left[2 \cdot q \cdot I_0 \cdot (1 + \cos(\phi_b)) \cdot R_D + \frac{[I_0 \cdot (1 + \cos(\phi_b)) \cdot R_D]^2}{\Delta f} + \frac{G_n^2 \cdot e_n^2}{R_f^2}\right]} = 0.006325$$

$$8.933 \cdot 0.482 \cdot 6.309 \cdot 0.871 \cdot (0.000267) = 0.006317$$

$R_D \quad 1.226 \qquad\qquad I_0 = 6 \times 10^{-6}$

Define functions for graphing

$$B_L(\omega_1,\omega_2) := \frac{\omega_1\cdot\omega_2}{4\cdot(\omega_1+\omega_2)}$$

$$c_{rw}(I_0,\phi_b,\omega_1,\omega_2) := \frac{180}{\pi}\cdot\frac{1}{(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D\cdot R_f\cdot G_E\cdot G_F\cdot ADC)}\cdot\tau\cdot\sqrt{\frac{3600}{\tau}}\cdot\sqrt{(ADC\cdot G_F)^2\cdot PSD_0(I_0,\phi_b)\cdot B_L(\omega_1,\omega_2)+\frac{1}{12}}$$

$$shot(I_0,\phi_b,\omega_1,\omega_2) := \frac{180}{\pi}\cdot\tau\cdot\sqrt{\frac{3600}{\tau}}\cdot\frac{\sqrt{(ADC\cdot G_F)^2\cdot\left[2\cdot q\cdot I_0\cdot(1+\cos(\phi_b))\cdot R_D\cdot R_f{}^2\cdot G_E{}^2\right]\cdot B_L(\omega_1,\omega_2)}}{(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D\cdot R_f\cdot G_E\cdot G_F\cdot ADC)}$$

$$excess(I_0,\phi_b,\omega_1,\omega_2) := \frac{180}{\pi}\cdot\tau\cdot\sqrt{\frac{3600}{\tau}}\cdot\frac{\sqrt{(ADC\cdot G_F)^2\cdot\left[\frac{\left[I_0\cdot(1+\cos(\phi_b))\cdot R_D\right]^2}{\Delta f}\cdot R_f{}^2\cdot G_E{}^2\right]\cdot B_L(\omega_1,\omega_2)}}{(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D\cdot R_f\cdot G_E\cdot G_F\cdot ADC)}$$

$$Rtherm(I_0,\phi_b,\omega_1,\omega_2) := \frac{180}{\pi}\cdot\tau\cdot\sqrt{\frac{3600}{\tau}}\cdot\frac{\sqrt{(ADC\cdot G_F)^2\cdot\left[\left(\frac{4\cdot k\cdot T_K}{R_f}\right)\cdot R_f{}^2\cdot G_E{}^2\right]\cdot B_L(\omega_1,\omega_2)}}{(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D\cdot R_f\cdot G_E\cdot G_F\cdot ADC)}$$

$$amp(I_0,\phi_b,\omega_1,\omega_2) := \frac{180}{\pi}\cdot\tau\cdot\sqrt{\frac{3600}{\tau}}\cdot\frac{\sqrt{(ADC\cdot G_F)^2\cdot\left[\left(i_n{}^2+\frac{G_n{}^2\cdot e_n{}^2}{R_f{}^2}\right)\cdot R_f{}^2\cdot G_E{}^2\right]\cdot B_L(\omega_1,\omega_2)}}{(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D\cdot R_f\cdot G_E\cdot G_F\cdot ADC)}$$

$$adc(I_0,\phi_b) := \frac{180}{\pi}\cdot\tau\cdot\sqrt{\frac{3600}{\tau}}\cdot\frac{\sqrt{\frac{1}{12}}}{(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D\cdot R_f\cdot G_E\cdot G_F\cdot ADC)}$$

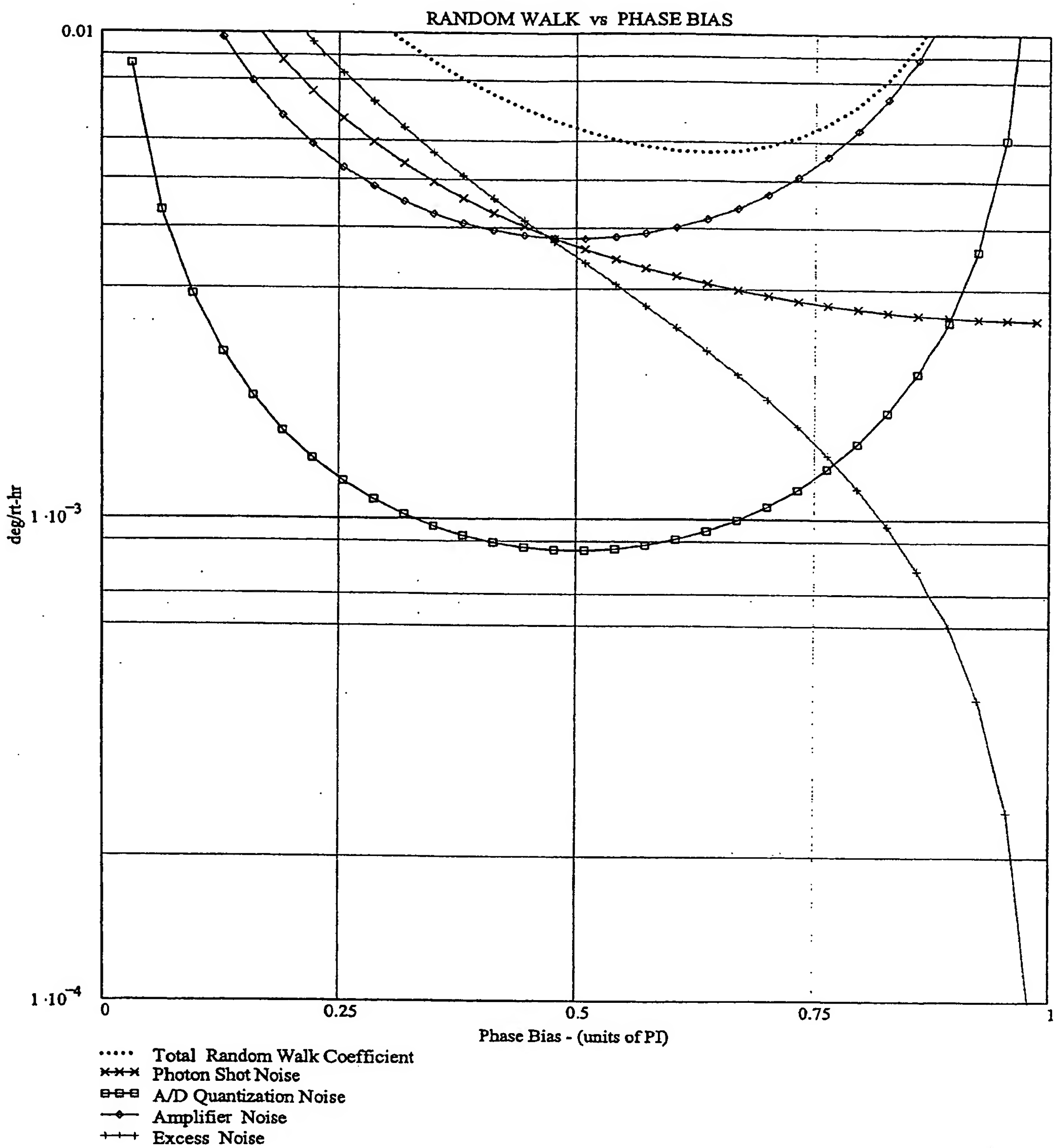
$$\phi_b := 0,.1..\pi$$

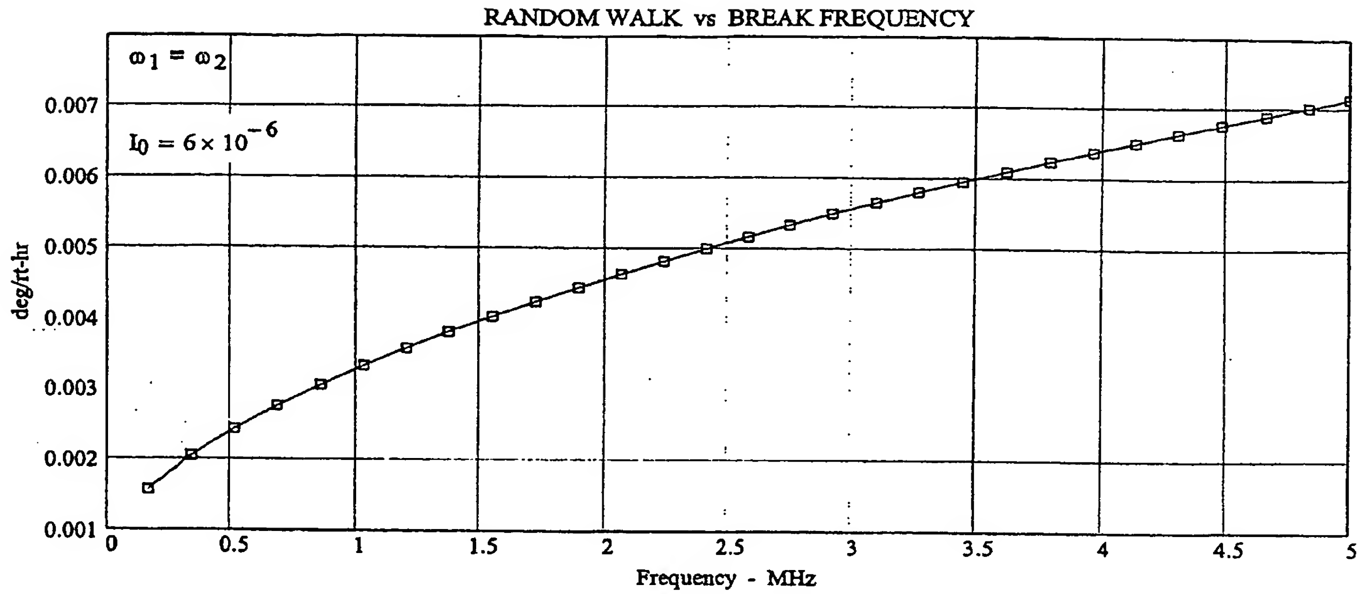
$$\sigma_{lsb}(I_0,\phi_b,\omega_1,\omega_2) := ADC\cdot G_F\cdot\sqrt{PSD_0(I_0,\phi_b)\cdot B_L(\omega_1,\omega_2)}$$ lsb rms at input to A/D

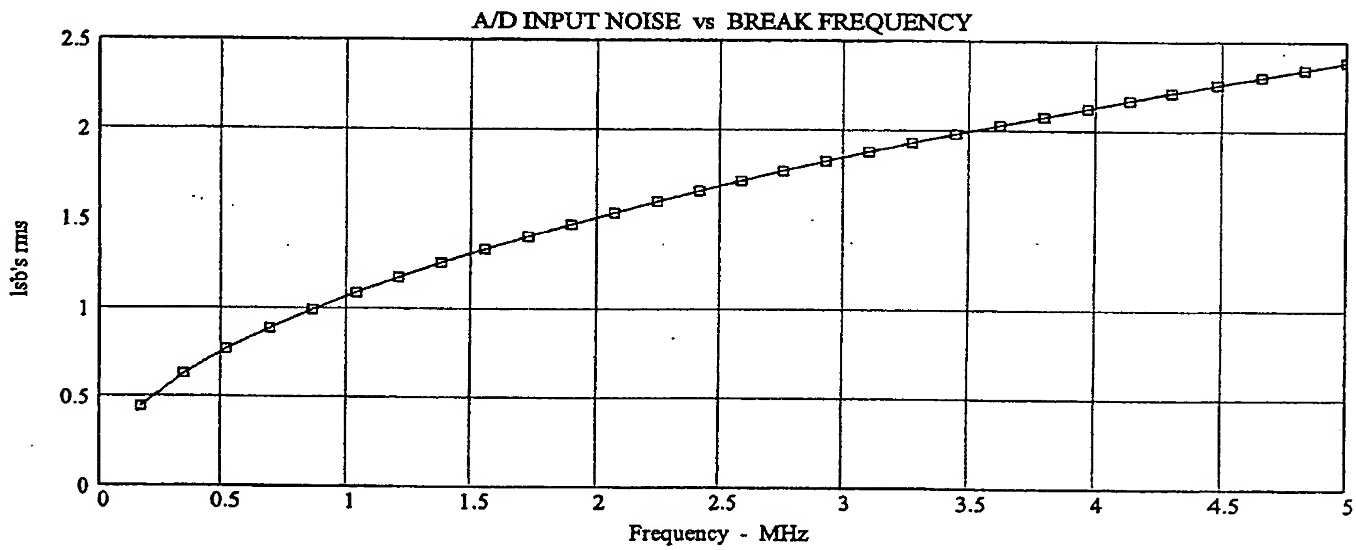
$$\sigma_\theta(I_0,\phi_b,\omega_1,\omega_2,T) := \frac{180}{\pi}\cdot\tau\cdot\sqrt{\frac{T}{\tau}}\cdot\frac{\sqrt{(ADC\cdot G_F)^2\cdot PSD_0(I_0,\phi_b)\cdot B_L(\omega_1,\omega_2)+\frac{1}{12}}}{(K_S\cdot I_0\cdot\sin(\phi_b)\cdot R_D\cdot R_f\cdot G_E\cdot G_F\cdot ADC)}$$

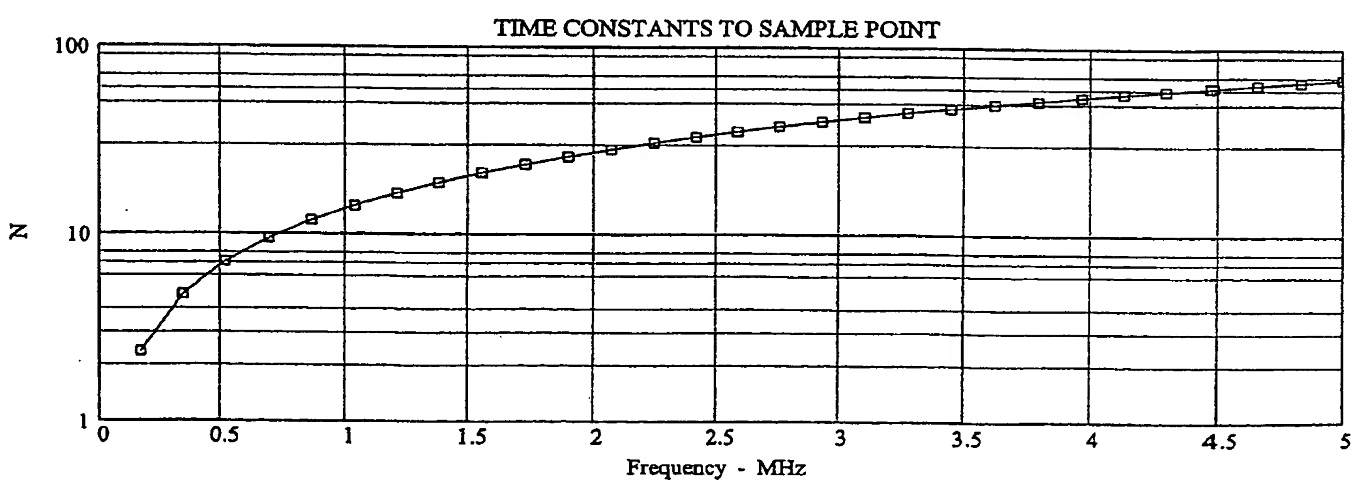
$$f := f_M, 2\cdot f_M .. 5\cdot 10^6$$

$\frac{\omega_1}{2 \cdot \pi} = 4.42 \times 10^6$. $\frac{\omega_2}{2 \cdot \pi} = 3.62 \times 10^6$ $I_0 = 6 \times 10^{-6}$.







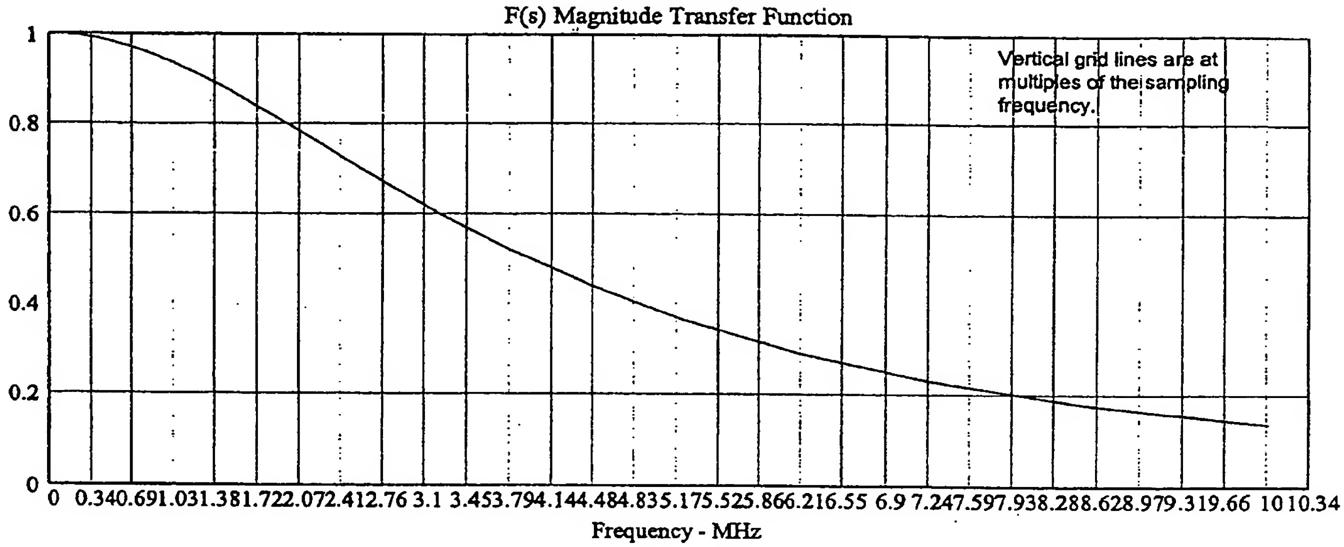


Note: boxes are at multiples of the modulation frequency.

Discussion of aliasing

$np := 100 \qquad i := 0..np \qquad f_i := 10^{\frac{i}{np}\cdot 7} \quad MFa(f) := |F(2\cdot\pi\cdot i\cdot f)| \qquad MFsa_i := MFa(f_i)$

$$\sqrt{\sum_{n=-100}^{100} \left(\left|F(2\cdot\pi\cdot i\cdot n\cdot f_s)\right|\right)^2} = 4.26$$

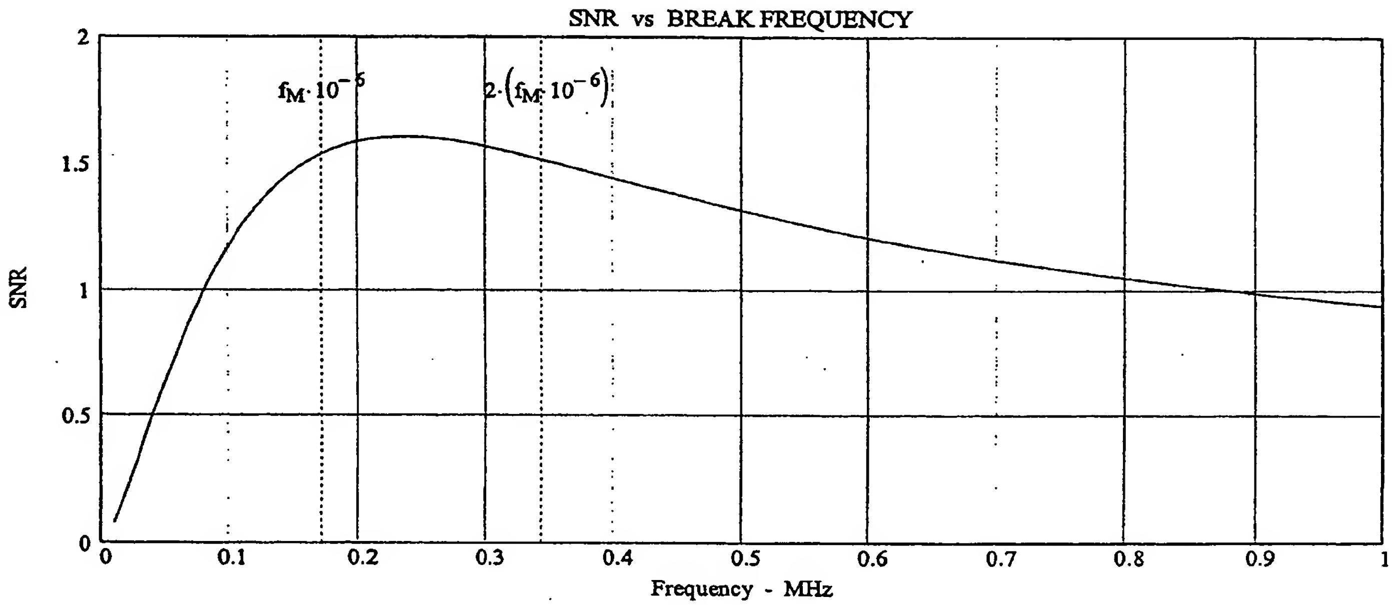


Optimization of SNR

Step function for $F(s)$ with $\omega_1 = \omega_2 = \omega_0$ $\quad S(t,\omega_0) := 1 - (1 + \omega_0 \cdot t) \cdot e^{-\omega_0 \cdot t}$

Signal-to-noise ratio: $\quad \mathrm{snr}(f_0) := \dfrac{S\left(\frac{3}{4} \cdot \tau, 2 \cdot \pi \cdot f_0\right)}{\sigma_{lsb}\left(I_0, \frac{\pi}{2}, 2 \cdot \pi \cdot f_0, 2 \cdot \pi \cdot f_0\right)}$

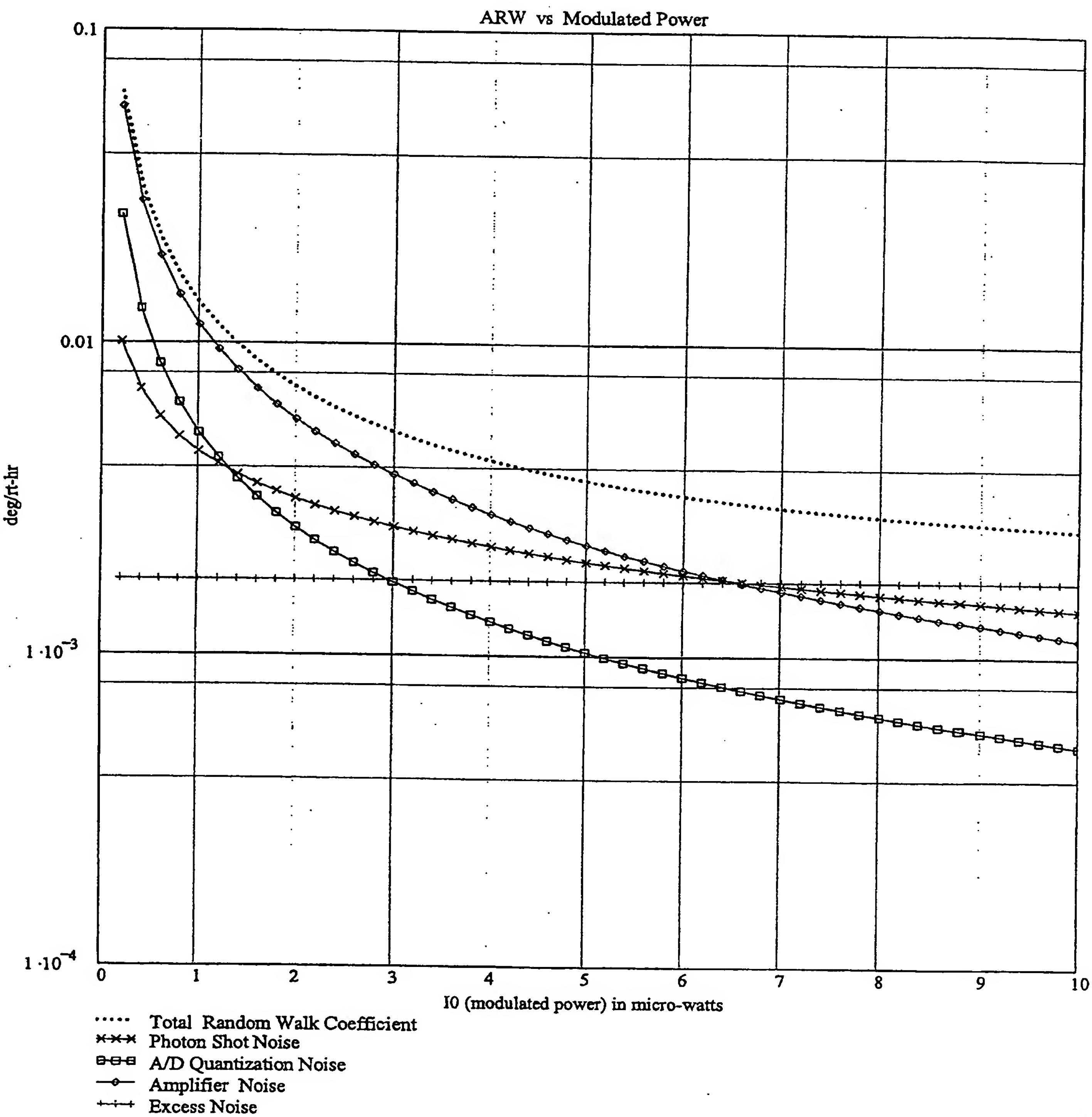
$f_0 := 10^4, 2 \cdot 10^4 .. 10^6$ $\quad I_0 = 6 \times 10^{-6}$



ARW vs I_0 plot (semilog plot)

Set phase and break frequencies for plot: $\phi_b := \frac{\pi}{2}$ $\omega_1 := 2 \cdot \pi \cdot 10^6$ $\omega_2 := 2 \cdot \pi \cdot 10^6$

Set maximum I_0 for plot: $I0_{max} := 10 \cdot 10^{-6}$ watts $np := 50$ $i := 1 .. np$ $I0_i := \frac{i}{np} \cdot I0_{max}$

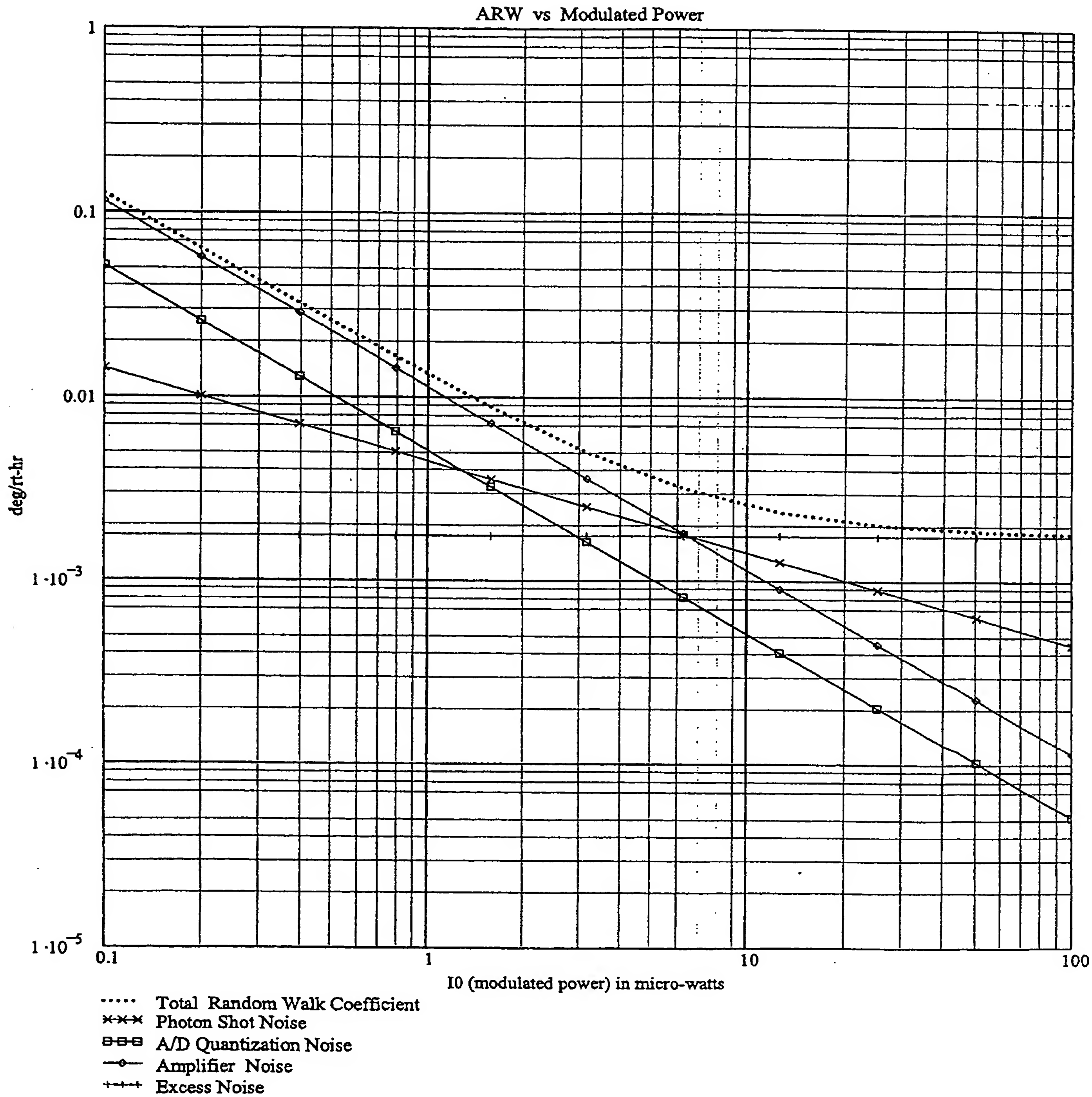


ARW vs I_0 plot (log-log plot)

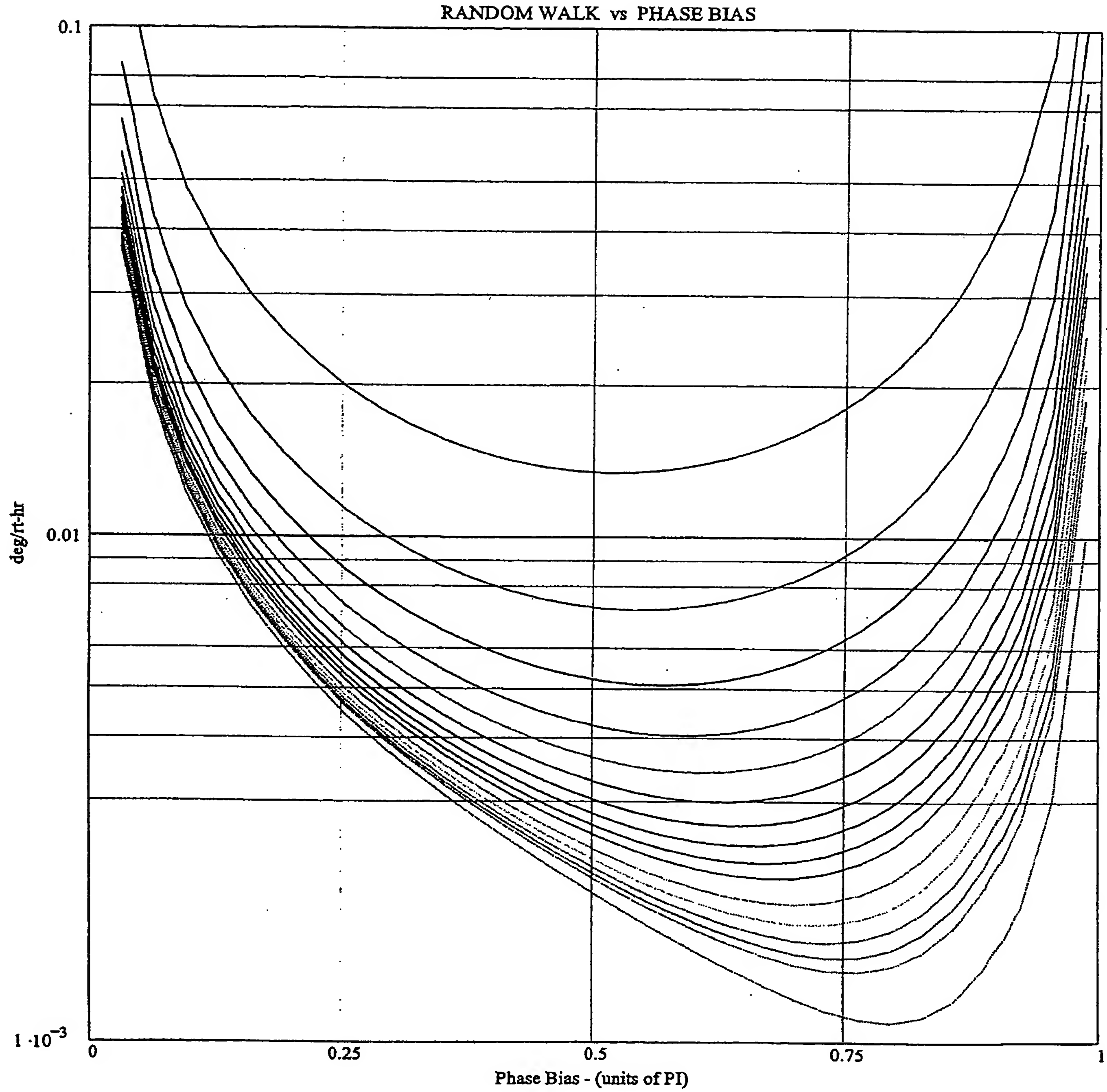
Set phase and break frequencies for plot: $\phi_b := \frac{\pi}{2}$ $\omega_1 := 2\cdot\pi\cdot 10^6$ $\omega_2 := 2\cdot\pi\cdot 10^6$

Set mimimum power level and number of power decades for plot: $I0_{min} := .1\cdot 10^{-6}$ $n\_decades := 3$

$np := 10$ $i := 0..np$ $I0_i := I0_{min}\cdot 10^{\frac{i}{np}\cdot n\_decades}$



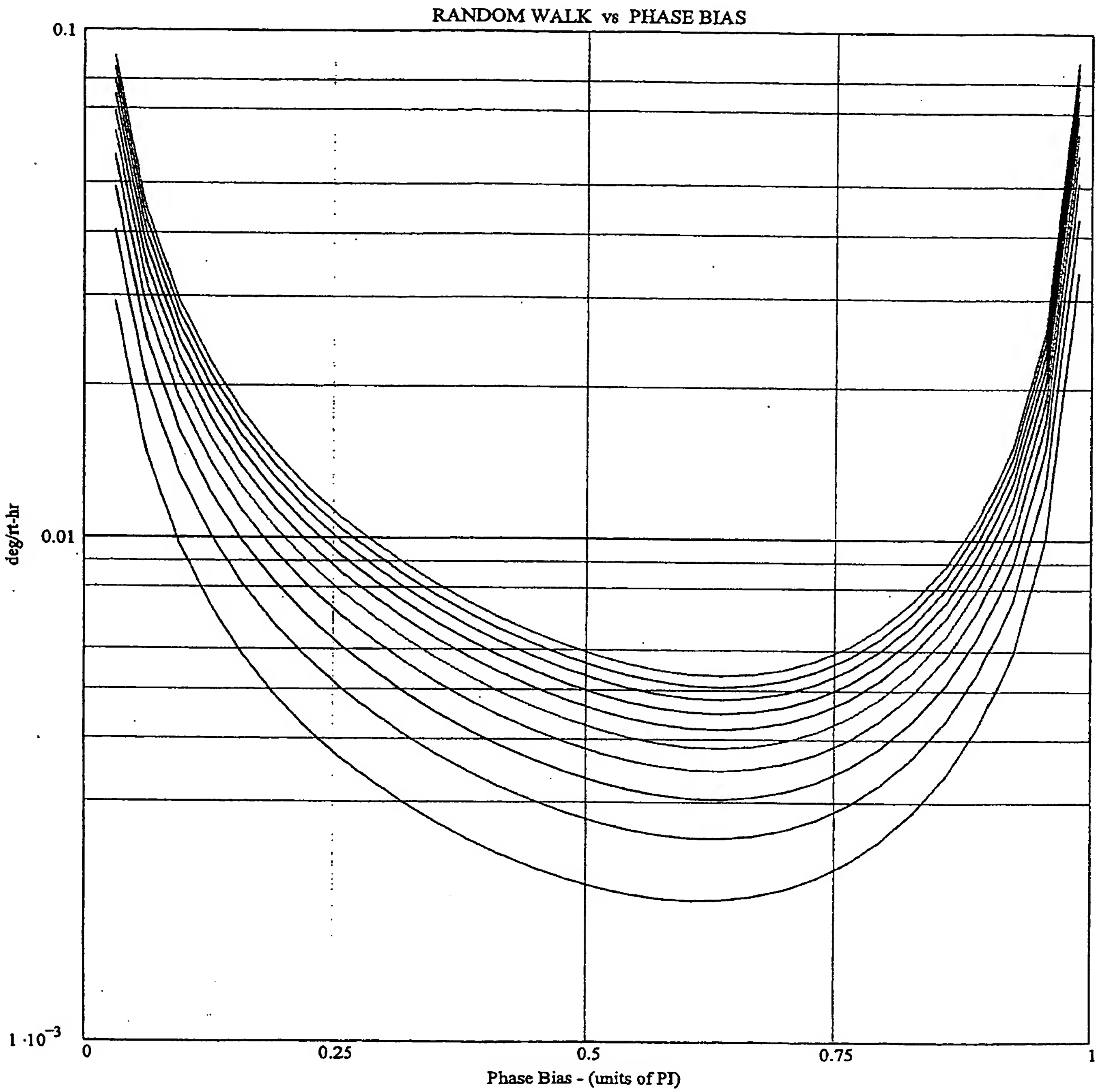
$\omega_1 := 2\cdot\pi\cdot 1\cdot 10^6$ $\omega_2 := 2\cdot\pi\cdot 1\cdot 10^6$ $I_1 := 10^{-6}$ $\phi_b := 0, .1 .. \pi$



Solid curves have (counting from top curve) I_0 = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 μwatt
Dotted curves have (counting from top) I_0 = 12, 14, 16, 18, 20, 30 μwatt

BEST AVAILABLE COPY

$\omega_1 := 2 \cdot \pi \cdot f_M$ $\omega_2 := \omega_1$ $I_0 := 6 \cdot 10^{-6}$ $\phi_b := 0, .1 .. \pi$



Solid curves have (counting from bottom curve) $\omega_1 = \omega_2 = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 \times \omega_M$
where ω_M is the modulation frequency

ANTI-ALIASING FILTER BANDWIDTH TRIM TABLE

Set modulation bias: $\phi_b := \frac{\pi}{2}$ Set desired rms noise at A/D: $rms\_noise := 0.8$ lsb

Set lower limit for filter corner frequency as multiple of modulation frequency: $lower := 2$

(For larger values of I_0 the computed value of the filter corner frequency may be too low, according to some criterion other than rms noise, like settling time etc. Setting this parameter defines the lower limit.)

Compute table

$i_I := 1 .. 24$ $f := 10^6$ $f\_set_{i_I} := root\left(\sigma_{lsb}\left(i_I \cdot 0.5 \cdot 10^{-6}, \phi_b, 2\cdot\pi\cdot f, 2\cdot\pi\cdot f\right) - rms\_noise, f\right)$

$f\_set_{i_I} := if\left[\left(\frac{lower}{2\cdot\tau} > \left|f\_set_{i_I}\right|\right) + \left(Im\left(f\_set_{i_I}\right) \ne 0\right), \frac{lower}{2\cdot\tau}, f\_set_{i_I}\right]$ $OUT_{i_I-1,0} := i_I \cdot 0.5$ $OUT_{i_I-1,1} := f\_set_{i_I} \cdot 10^{-6}$

OUT =

I_0 (μW)	f_{corner} (MHz)
0.5	1.438
[illegible]	[illegible]
2.5	1.018
3.5	0.854
4	0.784
4.5	0.719
5	0.662
5.5	0.61
6	0.564
6.5	0.522
7	0.485
7.5	0.449
8	0.418
8.5	0.39
9	0.364
9.5	0.345
10	0.345
10.5	0.345
11	0.345
11.5	0.345
12	0.345

I_0 is the modulated detector power

f_{corner} is the corner frequency of a two stage filter with the same corner frequency in each stage

$$\sigma_{lsb}\left[I_0, \phi_b, 2\cdot(2\cdot\pi\cdot f_M), 2\cdot(2\cdot\pi\cdot f_M)\right] = 0.626$$

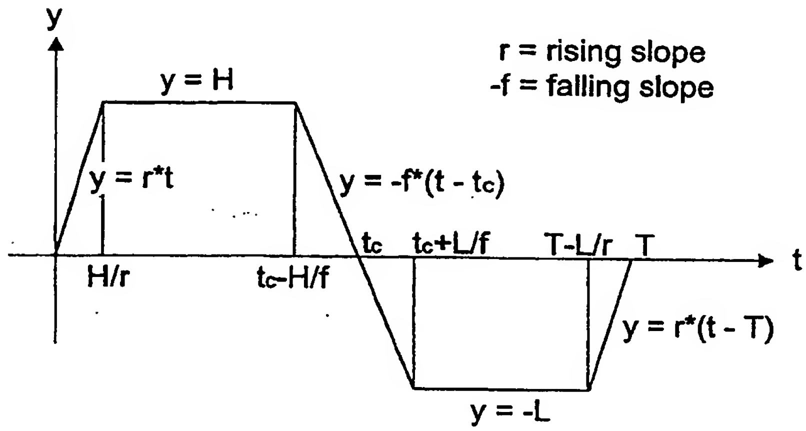
$$\sigma_{lsb}\left[\frac{I_0}{2}, \phi_b, 2\cdot(2\cdot\pi\cdot f_M), 2\cdot(2\cdot\pi\cdot f_M)\right] = 0.487$$

$$\sigma_{lsb}\left[I_0, \phi_b, 3\cdot(2\cdot\pi\cdot f_M), 3\cdot(2\cdot\pi\cdot f_M)\right] = 0.766$$

$$\frac{2\cdot f_M}{10^6} = 0.345$$

HARMONICS OF AN INPERFECT MODULATION SIGNAL

MODULATION SIGNAL



Harmonics of the modulation signal shown in the figure will be derived. The signal differs from a perfect square wave modulation in three ways:

1. Finite, unequal rise and fall times
2. Duty_cycle ≠ 50%
3. Unequal high and low magnitudes

Use the following transform-inverse pair:

$$h(n) = \frac{1}{T}\cdot\int_0^T \exp(i\cdot n\cdot\omega_0\cdot t)\cdot F(t)\,dt \qquad F(t) = \sum_{n=-\infty}^{\infty} h(n)\cdot\exp(-i\cdot n\cdot\omega_0\cdot t) \qquad \omega_0 := 2\cdot\pi\cdot\frac{1}{T}$$

Examples:

$$\frac{1}{T}\cdot\int_0^T \exp(i\cdot n\cdot\omega_0\cdot t)\cdot\sin(\omega_0\cdot t)\,dt = \frac{n\cdot i}{2}$$ for (n = +/-1), = 0 otherwise

$$\frac{1}{T}\cdot\int_0^T \exp(i\cdot n\cdot\omega_0\cdot t)\cdot\cos(\omega_0\cdot t)\,dt = \frac{1}{2}$$ for (n = +/-1), = 0 otherwise

$$\frac{1}{T}\cdot\int_0^T \exp(i\cdot n\cdot\omega_0\cdot t)\cdot\exp(i\cdot m\cdot\omega_0\cdot t)\,dt = \delta(n,-m)$$

The transform of the modulation function shown in the figure is:

$$\mathrm{HARn}(T,t_c,r,f,H,L,n) = \frac{1}{T}\cdot\int_0^{\frac{H}{r}} \exp(i\cdot n\cdot\omega_0\cdot t)\cdot(r\cdot t)\,dt \ldots$$

$$+\frac{1}{T}\cdot\int_{\frac{H}{r}}^{t_c-\frac{H}{f}} \exp(i\cdot n\cdot\omega_0\cdot t)\cdot(H)\,dt \ldots$$

$$+\frac{1}{T}\cdot\int_{t_c-\frac{H}{f}}^{t_c+\frac{L}{f}} \exp(i\cdot n\cdot\omega_0\cdot t)\cdot\left[-f\cdot(t-t_c)\right]\,dt \ldots$$

$$+\frac{1}{T}\cdot\int_{t_c+\frac{L}{f}}^{T-\frac{L}{r}} \exp(i\cdot n\cdot\omega_0\cdot t)\cdot(-L)\,dt \ldots$$

$$+\frac{1}{T}\cdot\int_{T-\frac{L}{r}}^{T} \exp(i\cdot n\cdot\omega_0\cdot t)\cdot[r\cdot(t-T)]\,dt$$

Carrying out the integral gives the general expression:

$$\mathrm{HARn}(T,t_c,r,f,H,L,n) := \frac{-1}{4\cdot n^2\cdot\pi^2}\cdot\left[\begin{array}{l} r\cdot T\cdot\left[\exp\left[-2i\cdot n\cdot\pi\cdot\frac{(-T\cdot r+L)}{(T\cdot r)}\right]-\exp\left[2\cdot i\cdot n\cdot\pi\cdot\frac{H}{(T\cdot r)}\right]\right]\ldots \\ +f\cdot T\cdot\left[\exp\left[2\cdot i\cdot n\cdot\pi\cdot\frac{(t_c\cdot f+L)}{(T\cdot f)}\right]-\exp\left[-2i\cdot n\cdot\pi\cdot\frac{(-t_c\cdot f+H)}{(T\cdot f)}\right]\right]\end{array}\right]$$

For n = 0, this is: $\mathrm{HARn}(T,t_c,r,f,H,L,0) = \frac{-1}{2}\cdot\frac{(H^2-L^2)\cdot(r+f)}{f\cdot r\cdot T}+(L+H)\cdot\frac{t_c}{T}-L$

For r, f --> ∞: $\mathrm{HARn\_ir}(T, t_c, H, L, n) := \frac{1}{n \cdot \pi} \cdot (L + H) \cdot e^{\left(i \cdot \pi \cdot n \cdot \frac{t_c}{T}\right)} \cdot \sin\left(n \cdot \pi \cdot \frac{t_c}{T}\right)$

For n = 0, this is: $\frac{(H + L)}{T} \cdot t_c - L$

which gives the Fourier representation:

$$M(t) = (L + H) \cdot \frac{t_c}{T} - L + \frac{2}{\pi} \cdot (L + H) \cdot \sum_{n=1}^{\infty} \frac{\sin\left(n \cdot \pi \cdot \frac{t_c}{T}\right)}{n} \cdot \cos\left[n \cdot \omega_0 \cdot \left(t - \frac{1}{2} \cdot t_c\right)\right]$$

(instantaneous rise and fall)

Note, that for $t_c = \frac{1}{2} \cdot T$ (50% duty cycle), this reduces to:

$$M(t) = \frac{H - L}{2} + \frac{2}{\pi} \cdot (L + H) \cdot \sum_{\substack{n=1 \\ (n\ \text{odd})}}^{\infty} \frac{1}{n} \cdot \sin(n \cdot \omega_0 \cdot t)$$

(for the case where $L \neq H$ is the only defect there are no even harmonics)

EVEN HARMONICS FROM DUTY CYCLE VARIATION

If L = H = A in addition to istantaneous rise and fall, the representation becomes:

$$M_\delta(t) = 2\cdot A\cdot\left(\frac{t_c}{T}-\frac{1}{2}\right)+\frac{4\cdot A}{\pi}\cdot\sum_{n=1}^{\infty}\frac{\sin\left(n\cdot\pi\cdot\frac{t_c}{T}\right)}{n}\cdot\cos\left[n\cdot\omega_0\cdot\left(t-\frac{1}{2}\cdot t_c\right)\right]$$

(Duty_cycle ≠ 50% is the only defect here)

or

$$M_\delta(t) = 2\cdot A\cdot\delta+\frac{4\cdot A}{\pi}\cdot\sum_{n=1}^{\infty}\frac{\sin(n\cdot\pi\cdot D)}{n}\cdot\cos\left[n\cdot\omega_0\cdot\left(t-\frac{1}{2}\cdot D\cdot T\right)\right]$$

where $t_c = T\cdot D$ $\quad D = \frac{1}{2}+\delta$ $\quad \delta = D-\frac{1}{2}$ (D is the duty cycle, D = 0.5 for 50%)

Picking out the even harmonics:

$$M_{\delta even}(t) = 2\cdot A\cdot\delta+\frac{2\cdot A}{\pi}\cdot\sum_{k=1}^{\infty}\frac{\sin(2\cdot k\cdot\pi\cdot\delta)}{k}\cdot\cos(2\cdot k\cdot\omega_0\cdot t-2\cdot k\cdot\pi\cdot\delta)$$

The 2nd harmonic term is:

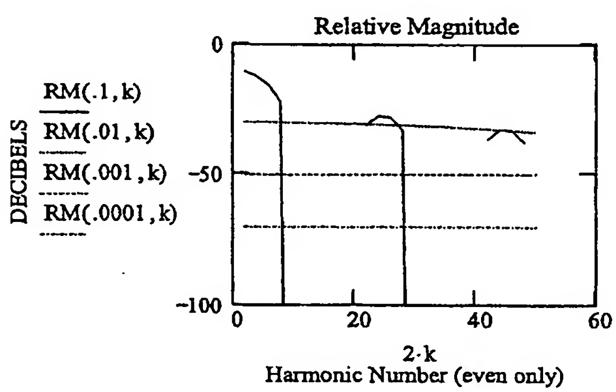
$$M_{\delta 2}(t) = 2\cdot\frac{A}{\pi}\cdot\sin(2\cdot\pi\cdot\delta)\cdot\cos(2\cdot\omega_0\cdot t-2\cdot\pi\cdot\delta)$$

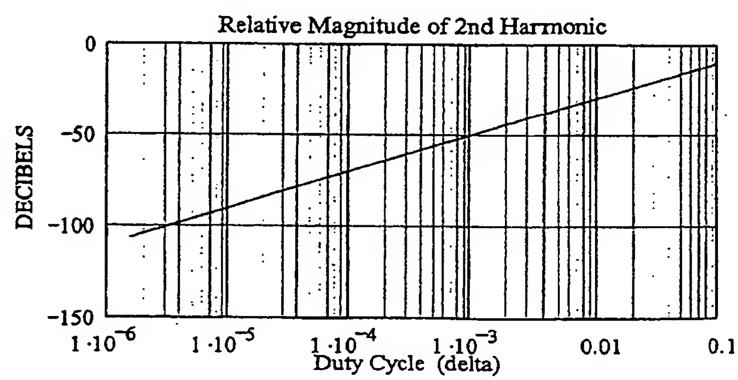
The even harmonic magnitude divided by fundamental magnitude is:

$$\text{Relative\_Magnitude}(n) = \frac{\frac{4\cdot A}{\pi}\cdot\frac{\sin(n\cdot\pi\cdot\delta)}{n}}{\frac{4\cdot A}{\pi}\cdot\sin(\pi\cdot D)} = \frac{\sin(n\cdot\pi\cdot\delta)}{n\cdot\sin(\pi\cdot D)}$$

(n even)

$$RM(\delta,k) := 20\cdot\log\left[\frac{\sin(2\cdot k\cdot\pi\cdot\delta)}{2\cdot k\cdot\sin\left[\pi\cdot\left(\frac{1}{2}+\delta\right)\right]}\right] \qquad k := 1..25 \qquad \delta\delta_k := 10^{-6}\cdot 10^{\frac{k}{25}\cdot 5}$$





For δ << 1: $\quad M_{\delta 2}(t) = 4\cdot A\cdot\delta\cdot\cos(2\cdot\omega_0\cdot t-2\cdot\pi\cdot\delta)$

$$\text{Relative\_Magnitude}(n) = \pi\cdot\delta$$

(n = 2, 4, 6, ...)

WAVEFORMS

Numerical study of even harmonics due to Duty_cycle ≠ 50%:

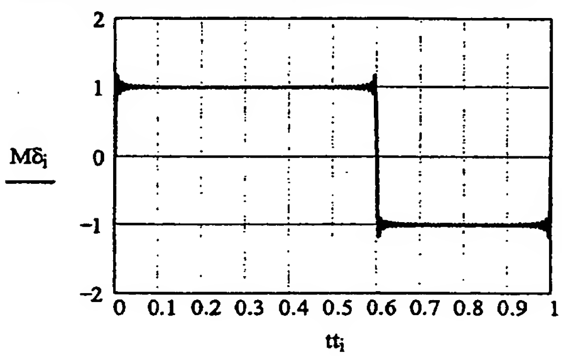
$A := 1 \quad D := .6 \quad \delta := D - \frac{1}{2} \quad T := 1 \quad \omega_0 := 2\cdot\pi\cdot\frac{1}{T} \quad N := 1000 \quad i := 0..N \quad tt_i := \frac{i}{N}\cdot T$

$$M_\delta(t) := 2\cdot A\cdot\delta + \frac{4\cdot A}{\pi}\cdot\sum_{n=1}^{100}\frac{\sin(n\cdot\pi\cdot D)}{n}\cdot\cos\left[n\cdot\omega_0\cdot\left(t-\frac{1}{2}\cdot D\cdot T\right)\right] \qquad M\delta_i := M_\delta(tt_i)$$

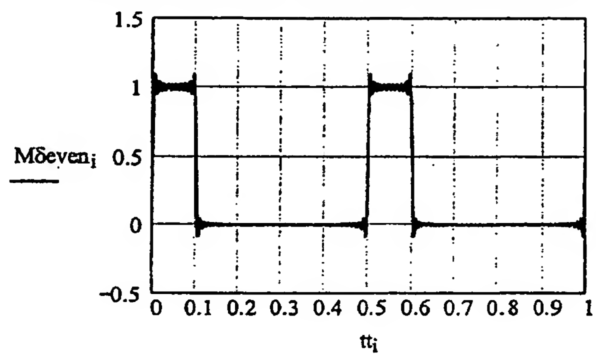
$$M_{\delta even}(t) := 2\cdot A\cdot\delta + \frac{2\cdot A}{\pi}\cdot\sum_{k=1}^{50}\frac{\sin(2\cdot k\cdot\pi\cdot\delta)}{k}\cdot\cos(2\cdot k\cdot\omega_0\cdot t - 2\cdot k\cdot\pi\cdot\delta) \qquad M\delta even_i := M_{\delta even}(tt_i)$$

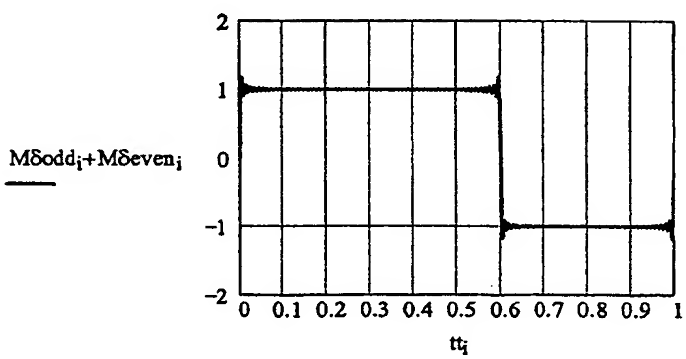
$$M_{\delta odd}(t) := \frac{4\cdot A}{\pi}\cdot\left[\sum_{k=0}^{50}\frac{\sin[(2\cdot k+1)\cdot\pi\cdot D]}{2\cdot k+1}\cdot\cos\left[(2\cdot k+1)\cdot\omega_0\cdot\left(t-\frac{1}{2}\cdot D\cdot T\right)\right]\right] \qquad M\delta odd_i := M_{\delta odd}(tt_i)$$

D = 60% signal reconstructed from Fourier series

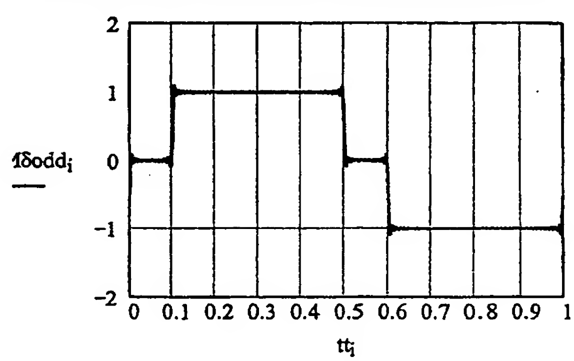


Signal constructed from even harmonics only





Signal constructed from odd harmonics only



EVEN HARMONICS DUE TO UNEQUAL RISE AND FALL TIMES

Setting L = H = A and $t_c = \frac{1}{2}\cdot T$ in the general formula, gives the harmonics for the case where th only defect in the modulation signal is unequal rise and fall times:

$$\mathrm{HARn}(T,r,f,A,n) = \frac{-1}{4}\cdot T\cdot\frac{r\cdot\exp\left[2i\cdot n\cdot\pi\cdot\frac{(r\cdot T-A)}{(r\cdot T)}\right]-r\cdot\exp\left[2i\cdot n\cdot\pi\cdot\frac{A}{(r\cdot T)}\right]+f\cdot\exp\left[i\cdot n\cdot\pi\cdot\frac{(f\cdot T+2\cdot A)}{(f\cdot T)}\right]-f\cdot\exp\left[i\cdot n\cdot\pi\cdot\frac{(f\cdot T-2\cdot A)}{(f\cdot T)}\right]}{\left(n^2\cdot\pi^2\right)}$$

and for n = 0: $\mathrm{HARn}(T,r,f,A,0) = 0$

This expression can also be written:

$$\mathrm{HARn}(T,r,f,A,n) = \frac{T\cdot i}{2\cdot n^2\cdot\pi^2}\cdot\left[r\cdot\sin\left(2\cdot n\cdot\pi\cdot A\cdot\frac{r^{-1}}{T}\right)-f\cdot(-1)^n\cdot\sin\left(2\cdot n\cdot\pi\cdot A\cdot\frac{f^{-1}}{T}\right)\right]$$

For r = f, this reduces to: $\mathrm{HARn}(T,r,r,A,n) = \frac{T\cdot i\cdot\left[1-(-1)^n\right]}{2\cdot n^2\cdot\pi^2}\cdot r\cdot\sin\left(2\cdot n\cdot\pi\cdot A\cdot\frac{r^{-1}}{T}\right)$

Introducing $t_{rise} = \frac{2\cdot A}{r}$ $t_{fall} = \frac{2\cdot A}{f}$ the general formula becomes:

$$\mathrm{HARn}(T,r,f,A,n) = \frac{A\cdot i}{n^2\cdot\pi^2}\cdot\left[\frac{T}{t_{rise}}\cdot\sin\left(n\cdot\pi\cdot\frac{t_{rise}}{T}\right)-(-1)^n\cdot\frac{T}{t_{fall}}\cdot\sin\left(n\cdot\pi\cdot\frac{t_{fall}}{T}\right)\right]$$

This formula can be used to derive the following series for the even and odd harmonics separately:

$$M_{even}(t) = \frac{A}{2\cdot\pi^2}\cdot\sum_{k=1}^{\infty}\frac{1}{k^2}\cdot\left(\frac{T}{t_{rise}}\cdot\sin\left(2\cdot k\cdot\pi\cdot\frac{t_{rise}}{T}\right)-\frac{T}{t_{fall}}\cdot\sin\left(2\cdot k\cdot\pi\cdot\frac{t_{fall}}{T}\right)\right)\cdot\sin\left(2\cdot k\cdot\omega_0\cdot t\right)$$

$$M_{odd}(t) = \frac{2\cdot A}{\pi^2}\cdot\sum_{k=0}^{\infty}\frac{1}{(2\cdot k+1)^2}\cdot\left[\frac{T}{t_{rise}}\cdot\sin\left[(2\cdot k+1)\cdot\pi\cdot\frac{t_{rise}}{T}\right]+\frac{T}{t_{fall}}\cdot\sin\left[(2\cdot k+1)\cdot\pi\cdot\frac{t_{fall}}{T}\right]\right]\cdot\sin\left[(2\cdot k+1)\cdot\omega_0\cdot t\right]$$

Expanding in the small parameters r^{-1} and f^{-1} (or equivalently t_{rise}, and t_{fall}) gives the even harmonics:

$$\mathrm{HARn}(T,r,f,A,n\_even) = \frac{i\cdot n\cdot\pi\cdot A}{6\cdot T^2}\cdot\left[\frac{(2\cdot A)^2}{f^2}-\frac{(2\cdot A)^2}{r^2}\right] = \frac{i\cdot n\cdot\pi\cdot A}{6}\cdot\left(\frac{t_{fall}^2-t_{rise}^2}{T^2}\right)$$

(small r^{-1} and f^{-1} or small t_{rise} and t_{fall})

which gives the representation:

$$M_{even}(t) = \frac{\pi\cdot A}{3}\cdot\left(\frac{t_{fall}^2-t_{rise}^2}{T^2}\right)\cdot\sum_{k=1}^{\infty}2\cdot k\cdot\sin\left(2\cdot k\cdot\omega_0\cdot t\right)$$

(small r^{-1} and f^{-1})

WAVEFORMS

$T := 1 \quad A := 1 \quad t_{rise} := .1 \quad t_{fall} := .2 \quad \omega_0 := 2 \cdot \frac{\pi}{T}$

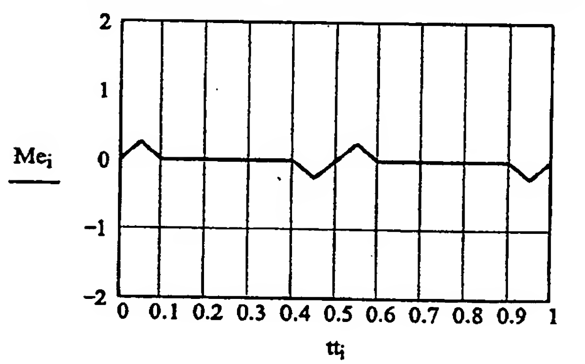
$$M_{even}(t) := \frac{A}{2\cdot\pi^2} \cdot \sum_{k=1}^{100} \frac{1}{k^2} \cdot \left(\frac{T}{t_{rise}} \cdot \sin\left(2\cdot k\cdot\pi\cdot\frac{t_{rise}}{T}\right) - \frac{T}{t_{fall}} \cdot \sin\left(2\cdot k\cdot\pi\cdot\frac{t_{fall}}{T}\right)\right) \cdot \sin(2\cdot k\cdot\omega_0\cdot t)$$

$$M_{odd}(t) := \frac{2\cdot A}{\pi^2} \cdot \sum_{k=0}^{100} \frac{1}{(2\cdot k+1)^2} \cdot \left[\frac{T}{t_{rise}} \cdot \sin\left[(2\cdot k+1)\cdot\pi\cdot\frac{t_{rise}}{T}\right] + \frac{T}{t_{fall}} \cdot \sin\left[(2\cdot k+1)\cdot\pi\cdot\frac{t_{fall}}{T}\right]\right] \cdot \sin[(2\cdot k+1)\cdot\omega_0\cdot t]$$

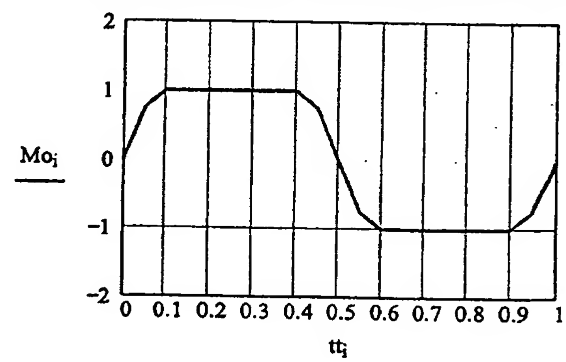
$N := 1000 \quad i := 0..N \quad tt_i := \frac{i}{N} \cdot T$

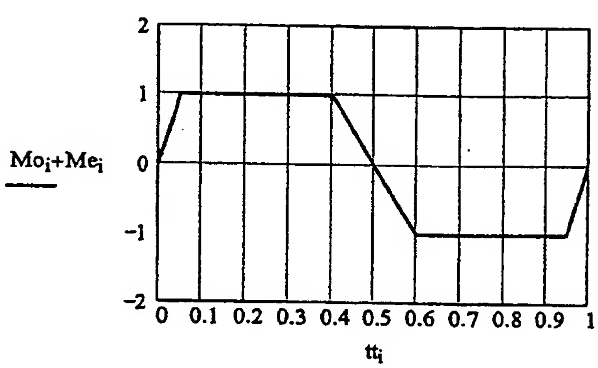
$Me_i := M_{even}(tt_i) \quad Mo_i := M_{odd}(tt_i)$

Signal constructed from even harmonics only



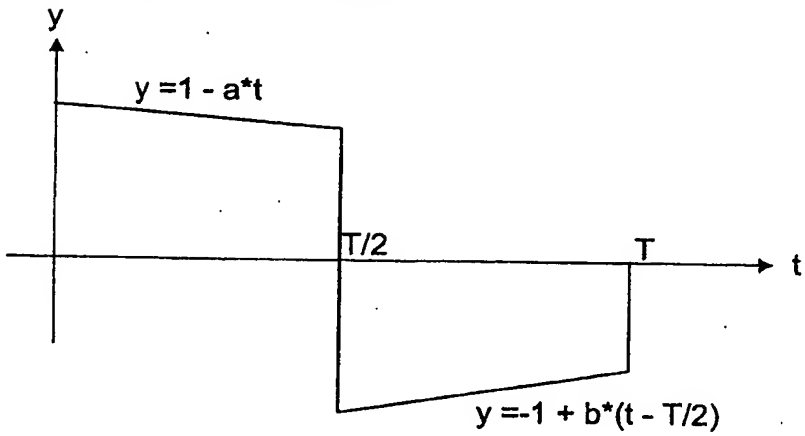
Signal constructed from odd harmonics only





HARMONICS OF A MODULATION SIGNAL WITH ASYMMETRIC DROOP

MODULATION SIGNAL



Harmonics of the modulation signal shown in the figure will be derived. The signal differs from a perfect square wave modulation by linear droop shown in th figure, where the droop slopes are, in general, not equal: $a \neq b$.

Use the following transform-inverse pair:

$$h(n) = \frac{1}{T}\cdot\int_0^T \exp(i\cdot n\cdot\omega_0\cdot t)\cdot F(t)\,dt \qquad F(t) = \sum_{n=-\infty}^{\infty} h(n)\cdot\exp(-i\cdot n\cdot\omega_0\cdot t) \qquad \omega_0 := 2\cdot\pi\cdot\frac{1}{T}$$

The transform of the modulation function shown in the figure is:

$$HARn(T,a,b,n) = \frac{1}{T}\cdot\int_0^{\frac{T}{2}} \exp(i\cdot n\cdot\omega_0\cdot t)\cdot(1-a\cdot t)\,dt\,\ldots$$
$$+\frac{1}{T}\cdot\int_{\frac{T}{2}}^{T} \exp(i\cdot n\cdot\omega_0\cdot t)\cdot\left[-1+b\cdot\left(t-\frac{T}{2}\right)\right]dt$$

Carrying out the integral gives the general expression:

$$HARn(T,a,b,n) = \frac{-i}{4\cdot n^2\cdot\pi^2}\cdot\left[\left[4\cdot n\cdot\pi\cdot\left[(-1)^n-1\right]-i\cdot T\cdot(a+b)\cdot\left[(-1)^n-1\right]\right]-n\cdot\pi\cdot T\cdot\left[a\cdot(-1)^n-b\right]\right]$$

For even n this is: $$HARn(T,a,b,n\_even) = \frac{i}{4\cdot n\cdot\pi}\cdot T\cdot(a-b) = \frac{i}{2\cdot n\cdot\pi}\cdot\left(a\cdot\frac{T}{2}-b\cdot\frac{T}{2}\right)$$

which giv s the following Fourier series for the even harmonic part of the modulation function:

$$M_{even}(t) = \frac{1}{\pi}\cdot\left(a\cdot\frac{T}{2} - b\cdot\frac{T}{2}\right)\cdot\sum_{n}\frac{1}{n}\cdot\sin(n\cdot\omega_0\cdot t)$$

n=2, 4, ...

If the nominal magnitude were +/- V_0, then the even harmonic magnitude relative to V_0 is:

$$\text{Relative\_Magnitude}(n) = \frac{\frac{1}{n\cdot\pi}\cdot\left(a\cdot V_0\cdot\frac{T}{2} - b\cdot V_0\cdot\frac{T}{2}\right)}{V_0} = \frac{1}{n\cdot\pi}\cdot\frac{\Delta V}{V_0}$$

where $\Delta V = a\cdot V_0\cdot\frac{T}{2} - b\cdot V_0\cdot\frac{T}{2}$ is the differential voltage droop.

Using $20\cdot\log\left(\frac{1}{2\cdot\pi}\right) = -15.964$, the second harmonic can be expressed in dB as

$$\text{Relative\_Magnitude\_dB}(2) = 20\cdot\log\left(\frac{\Delta V}{V_0}\right) - 15.964\ \text{dB}$$

Derivation of closed form approximation of gyro bias error

1. The switch waveform is:

	positive section	negative section
	$1 - a\cdot t$	$-1 + b\cdot\left(t - \frac{1}{2}\cdot T\right)$

2. Delaying with a tuning error gives:

	positive section	negative section
before delay:	$1 - a\cdot t$	$-1 + b\cdot\left(t - \frac{1}{2}\cdot T\right)$

	positive section	negative section
after delay:	$1 - a\cdot\left(t - \frac{1}{2}\cdot T - \Delta\right)$	$-1 + b\cdot(t - T - \Delta)$

3. This is equivalent to:

	negative section	positive section
after delay:	$-1 + b\cdot(t - \Delta)$	$1 - a\cdot\left(t - \frac{1}{2}\cdot T - \Delta\right)$

4. Change sign of delayed signal:

	positive section	negative section
before delay:	$1 - a \cdot t$	$-1 + b \cdot \left(t - \frac{1}{2} \cdot T\right)$
after delay and sign change:	$1 - b \cdot (t - \Delta)$	$-1 + a \cdot \left(t - \frac{1}{2} \cdot T - \Delta\right)$

5. Now add the two signals and multiply by $\pi/4$:

positive section

$$\frac{\pi}{2} - \frac{\pi}{4} \cdot (a+b) \cdot t + \frac{\pi}{4} \cdot b \cdot \Delta$$

negative section

$$\frac{-\pi}{2} + \frac{\pi}{4} \cdot (a+b) \cdot \left(t - \frac{1}{2} \cdot T\right) - \frac{\pi}{4} \cdot a \cdot \Delta$$

6. The positive section will be sampled at $t = \frac{3}{8} \cdot T$, and the negative section at $t = \frac{7}{8} \cdot T$

positive section

$$\frac{\pi}{2} - \frac{\pi}{4} \cdot (a+b) \cdot \frac{3}{8} \cdot T + \frac{\pi}{4} \cdot b \cdot \Delta$$

negative section

$$\frac{-\pi}{2} + \frac{\pi}{4} \cdot (a+b) \cdot \frac{3}{8} \cdot T - \frac{\pi}{4} \cdot a \cdot \Delta$$

7. Interference in the detector gives the cosine of these signals, and demodulation and integration takes the difference of the cosines:

$$\mathrm{RECT} = \cos\left[\frac{\pi}{2} - \frac{\pi}{4} \cdot (a+b) \cdot \frac{3}{8} \cdot T + \frac{\pi}{4} \cdot b \cdot \Delta\right] - \cos\left[\frac{-\pi}{2} + \frac{\pi}{4} \cdot (a+b) \cdot \frac{3}{8} \cdot T - \frac{\pi}{4} \cdot a \cdot \Delta\right]$$

8. Expanding in a and b: $\mathrm{RECT} = \frac{\Delta \cdot \pi}{4} \cdot (a - b)$

9. Applying the Sagnac scale factor and converting to deg/hr:

$$\mathrm{BIAS} = \frac{1}{2} \cdot \left[\frac{\Delta \cdot \pi}{4} \cdot (a - b)\right] \cdot \frac{\lambda \cdot c}{2 \cdot \pi \cdot L \cdot D} \cdot \frac{180}{\pi} \cdot 3600$$